Question-01: A loan of \$10,000 is to be repaid throug 10 annual payments, with the first payment due one month after the loan is issued. The loan carries a nominal annual interest rate of 12% compounded monthly. Create an amortization schedule for this loan.

Solution: Given,

$P = 10{,}000$

Annual interest rate $= 12\%$

monthly interest rate $= \frac{0.12}{12} = 0.01$

$i_m = 0.01$

$(1+i) = (1+i_m)^m$.

$\Rightarrow i = (1+0.01)^{12} - 1$

$\Rightarrow i = 0.1268 = 12.68\%$

Amount of each payment,

$$A = P \times \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= 10{,}000 \times \frac{0.1268(1+0.1268)^{10}}{(1+0.1268)^{10} - 1}$$

$$= 1819.387$$

Amortization schedule for this loan:

Duration	Payment	Interest	Principle Repaid	Out... Pr...
0				10,0
1	1819.387	1268	551.387	9448
2	1819.387	1198.08	621.3028	8827
3	1819.387	1119.30	700.084	8127.2
4	1819.387	1030.5322	788.855	7338.3
5	1819.387	930.505	888.88	6449.49
6	1819.387	817.795	1001.59	5447.901
7	1819.387	690.7938	1128.593	4319.308
8	1819.387	547.688	1271.699	3047.609
9	1819.387	386.4368	1432.95	1614.65
10	1819.387	204.7387	1614.648	0

Question-02: sophia is planning her retirement savings and decides to invest in a special annuity this annuity provides her with quarterly payments of $25 for the first 5 years (20 payments in total), starting immediately. After this period, the annuity increases and begins paying $50 every quarter for the rest of her life, starting at the beginning of the 6th year.

Assuming an annual interest rate of 8%, compou[nded] quarterly.

(a) Calculate the present value of this annuity at the time of Sophia's investment.

Answer:

Given,

Nominal Rate = 8%

$$i_q = \frac{0.08}{4} = 0.02$$

In the 1st phase;

$25 per quarter for 5 years (20 payments in total) starting immediately → annuity due.

In the 2nd Phase,

$50 per quarter forever, starting at the beginning of year 6 (i.e. t = 5 years = 20th quarter) → perpetuity due.

Present value for the 1st phase will be-

$$PV_1 = \frac{1-v^n}{d} \times R$$

$$= \frac{1-(1+i)^{-n}}{1-v} \times R$$

$$= \frac{1-(1+i)^{-n}}{i} (1+i) \times R$$

$$= \frac{1-(1+0.02)^{-20}}{0.02} \times (1.02) \times 25$$

$$= 416.8865$$

Perpetuity due at year $t=5$,

$$PV_5 = \frac{50}{0.02}(1.02) = 2550$$

Discount back to present value.

$$PV_2 = 2550 \times (1.02)^{-20}$$

$$= 1716.0768$$

Therefore,

The present value of this annuity at the time of Sophia's investment will be-

$$PV = PV_1 + PV_2$$

$$= 416.8865 + 1716.0768$$

$$= 2132.9633$$

$$\simeq \$2133$$

Question-2(b): If Sophia instead had the option to receive a flat quarterly payment for life (starting immediately), what constant quarterly amount would give the same present value as the original annuity.

Answer:

Let, constant payment $= C$

As, Sophia receives a flat quarterly payment for life, then,

$$\text{Present value} = \frac{C}{i}(1+i)$$

$$\Rightarrow 2132.9633 = \frac{C}{0.02}(1+0.02)$$

$$\Rightarrow C = 41.8228$$

Therefore, if Sophia had the option to receive a flat quarterly payment for life (starting immediately), that would be \$41.82.

Question-02(c): Suppose, Sophia delays the purch[illegible] of the annuity by two years. What would be the new cost (present value at that time) for the same payment structure, assuming the interest rate remains unchanged?

Answer: As, Sophia delays the purchase of the annuity by two years (8 quarters).

Future value at $t = 2$

$$PV_{new} = 2132.9633(1.02)^8$$

$$= 2499.10[illegible]$$

If sophia waits 2 years, the same annuity becomes more expensive. Because money has time value the cost increases because the valuation is done later, so fewer discounting periods apply.

Question-03: Calculate the level premium P of 10-year endowment insurance on (40) of 1,000. Given that-

mortality: Use the illustrative life table

$i = 0.05$ and P is paid at the beginning of each year for 5 years.

Answer: A 10 year endowment insurance pays Death benefit if death occurs within 10 years or survival benefit if alive at end of 10 years.

For endowment insurance,

EPV (benefits) = Term insurance + Pure Endowment

Expected present value (benefits)

$= 1000 (A_{40:\overline{10|}} + v^{10} {}_{10}P_{40})$

where,

$$A_{40:\overline{10|}} = \sum_{K=1}^{10} v^K \; {}_{K-1}P_{40} \; q_{40+K-1}$$

Premiums are paid in advance for 5 years only if the person is alive. This is a 5-year temporary life annuity due.

Expected present value (premiums) $= P\ \ddot{a}_{40:\overline{5|}}$

$$= P \sum_{k=0}^{4} v^k \ {}_kP_{40}$$

Now, by equivalence principle,

EPV (premiums) = EPV (benefits)

$\Rightarrow P\ \ddot{a}_{40:\overline{5|}} = 1000\,(A_{40:\overline{10|}} + v^{10}\ {}_{10}P_{40})$

$$\Rightarrow P = \frac{1000\,(A_{40:\overline{10|}} + v^{10}\ {}_{10}P_{40})}{\ddot{a}_{40:\overline{5|}}}$$

This is the net level premium for a 10-year endowment insurance.

Question-04: An insurance company is offering a 10-year term life insurance policy to a 40-year old individual. The policy provides a death benefit of \$1,000 payable at the end of the year of death, if death occurs within the 10-year term.

You are given –

- Mortality: Use the illustrative life table
- Interest rate $i = 10\%$ annually.
- Premiums are payable annually in advance for 5 years, provided the insured is alive at the time of payment.
- The insurer incurs annual expenses of \$5 per policy (initial and renewal) and a one time initial expense of \$20 at policy issue.

Calculate the annual gross premium for the policy.

Answer:

Let, P be the annual gross premium

Expected present value of death benefits

$= 1000 \times A_{40:\overline{10|}}$

where,

$A_{40:\overline{10|}} = \sum_{k=1}^{10} v^k \; {}_{k-1}p_{40} \; q_{40+k-1}$

Total expenses:

Initial expense + Renewal expenses

$= 20 + 5\,\ddot{a}_{40:\overline{5|}}$

Expected present value (premium)

$= P\,\ddot{a}_{40:\overline{5|}}$

Now, by equivalence principle, we get -

$P\,\ddot{a}_{40:\overline{5|}} = 1000 \times A_{40:\overline{10|}} + (20 + 5 \times \ddot{a}_{40:\overline{5|}})$

$\Rightarrow P = \dfrac{1000\,A_{40:\overline{10|}} + 20 + 5\,\ddot{a}_{40:\overline{5|}}}{\ddot{a}_{40:\overline{5|}}}$

This is the annual gross premium for this policy.

Question-05: A life insurance company offers a retirement income annuity to a 40 year old individual. Under the plan:

- The policyholder will receive annual payments of \$1 for life, starting at age 65.
- No benefits is paid if the policyholder dies before age 65.
- The policyholder is required to pay annual premiums for 10 years, starting at age 40.
- After the first 5 years of premium payments the annual premium is reduced by 50% for the remaining 10 years.
- The ultimate age (maximum age) is 110.
- interest rate, $i = 0.05$

Determine the initial annual premium payable by the policyholder.

Answer: this is the example of deferred annuity.

Here, Deferral period = 25 years.

Expected present value of benefit will be-

$$EPV(B) = {}_{25}E_{40}\, \ddot{a}_{65}$$

$$= v^{25}\, {}_{25}p_{40}\, \ddot{a}_{65}$$

Premiums are paid only if the policyholder survives.

For first 5 years (0 to 4)

$$EPV_1 = P \sum_{k=0}^{4} v^k\, {}_kp_{40}$$

Answer:

For the next 5 years,

$$EPV_2 = 0.5\,P \sum_{K=5}^{9} v^K \,{}_KP_{40}$$

Total premium EPV will be -

$$EPV(P) = P\left[\sum_{K=0}^{4} v^K \,{}_KP_{40} + 0.5 \sum_{K=5}^{9} v^K \,{}_KP_{40}\right]$$

Now, by applying the equivalence principle, we get -

$$P\left[\sum_{K=0}^{4} v^K \,{}_KP_{40} + 0.5 \sum_{K=5}^{9} v^K \,{}_KP_{40}\right] = v^{25}\,{}_{25}P_{40}\, a_{65}$$

$$\Rightarrow P = \frac{v^{25}\,{}_{25}P_{40}\, a_{65}}{\sum_{K=0}^{4} v^K \,{}_KP_{40} + 0.5 \sum_{K=5}^{9} v^K \,{}_KP_{40}}$$

$$P = \frac{v^{25}\,{}_{25}P_{40}\, a_{65}}{a_{40:\overline{5}|} + 0.5\, v^5 \,{}_5P_{40}\, a_{45:\overline{5}|}}$$

This is the expression for the initial annual premium payable by the policyholder.

Question-06: Reconsider Question-05

The insurer charges:

- An initial expense of \$50, incurred at policy issue (age 40)
- A renewal expense of \$5, incurred at the start of each year that a premium is paid.

paid. Calculate the initial gross premium, assuming the present value of gross premiums equals the present value of benefits plus expenses.

Answer:

Expected present value of expenses = Initial expense + Renewal expenses.

$PVE = 50 + 5\, a_{40:\overline{5}|} + 5 v^5 \, {}_5P_{40}\, a_{45:\overline{5}|}$

Expected present value of premium =

$P_g \left[a_{40:\overline{5}|} + 0.5\, v^5 \, {}_5P_{40} \cdot a_{45:\overline{5}|} \right]$.

Now, by equivalence principle, we get–

$$P_g \left[a_{40:\overline{5}|} + 0.5\, v^5 \, {}_5P_{40}\, a_{45:\overline{5}|} \right] = v^{25}\, {}_{25}P_{40}\, a_{65} + 50 + 5\, a_{40:\overline{5}|} + 5 v^5 \, {}_5P_{40} \cdot a_{45:\overline{5}|}$$

$$\Rightarrow P_g = \frac{v^{25}\, {}_{25}P_{40}\, a_{65} + 50 + 5\, a_{40:\overline{5}|} + 5 v^5 \, {}_5P_{40}\, a_{45:\overline{5}|}}{a_{40:\overline{5}|} + 0.5\, v^5 \, {}_5P_{40}\, a_{45:\overline{5}|}}$$

This is the expression for the initial gross premium.

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