Mathematical Demography

Demography

Demography is the statistical study of human populations, examining their size, structure, and distribution over time and space. It analyzes changes in population dynamics through birth rates, death rates, and migration patterns.

Key Aspects of Demography:

- Population Size and Structure: Analyzing the number and composition of individuals in a population.
- Population Distribution: Studying how populations are spread across different geographic areas.
- Population Dynamics: Understanding changes in population numbers and structures over time due to births, deaths, and migration.

Core Elements of Demographic Study

Population Size Changes:

Analyzing increases or decreases in population numbers.

Population Structure:

Examining the composition based on age, sex, marital status, and ethnic origin.

Geographical Distribution:

Studying how populations are spread across different regions or territories.

Common Demographic Concepts

- **Fertility :** The capacity to produce offspring, often measured by birth rates.
- **Mortality:** The incidence of death within a population, typically quantified by death rates.
- **Migration:** The movement of individuals between locations, affecting population size and composition.
- **Marriage and Family Dynamics:** Patterns and trends in marriage, family size, and childbearing.
- **Reproductive Health:** Aspects related to contraception, sterility, and reproductive rights.
- **Social Mobility:** The ability of individuals or groups to move within a social hierarchy.

Mathematical Demography

Mathematical demography applies **mathematical models** and **techniques** to the study of population dynamics. It aims to understand how populations grow, decline, and shift in structure over time due to factors such as **fertility, mortality, and migration**.

Objectives of Mathematical Demography

- To analyze the growth and decline of populations.
- To forecast future population size and structure.
- To examine the effects of fertility, mortality, and migration on populations.
- To aid in policy-making related to healthcare, urban planning, and economics.

Age and Sex Composition

The age and sex composition of a population refers to how individuals are distributed across different age groups and genders. This data provides essential insights into population growth, the dependency ratio, and future needs in sectors such as healthcare, education, and employment.

Age Composition: This refers to the distribution of individuals across various age groups. It's a crucial determinant of the labor force, dependency ratio, and future population growth.

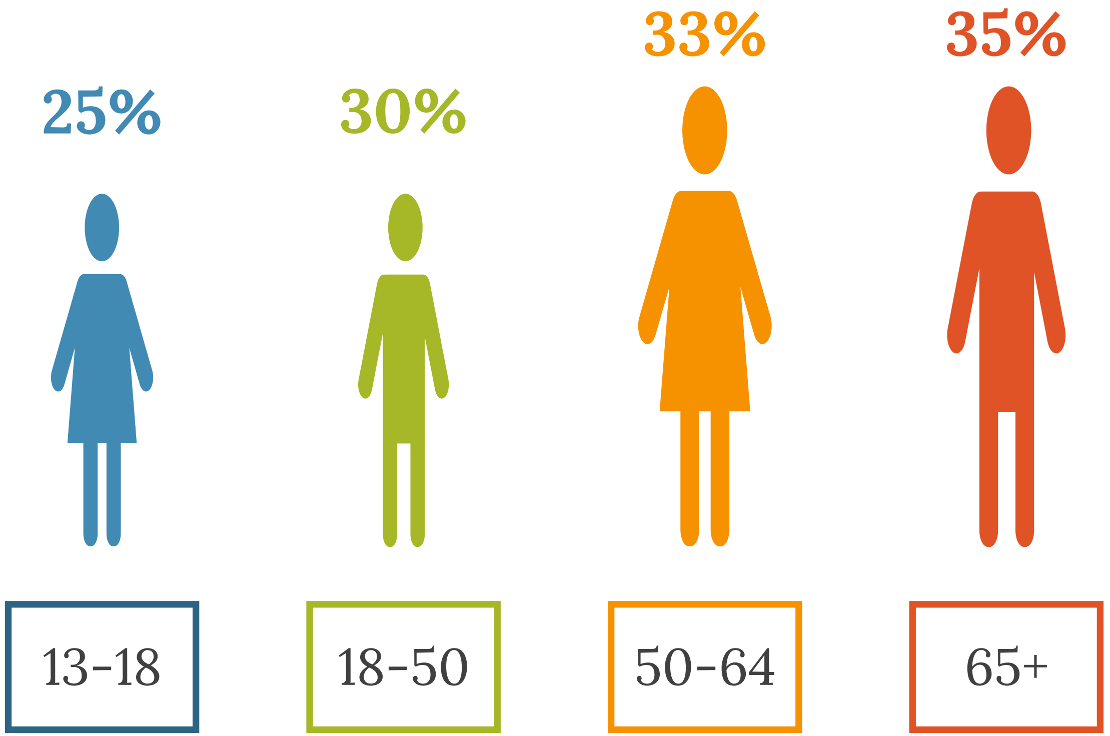
Sex Composition: This refers to the distribution of males and females in a population. The sex ratio is an important demographic indicator.

Age and Sex Composition

Age Distribution

80% Male

60% Female



Age Composition

Age composition refers to the distribution of individuals in a population across different age groups, such as children, working-age adults, and elderly.

The population is categorized into age groups:

- **Children (0-14 years):** Future potential workforce, high fertility rate.
- **Working-Age Adults (15-64 years):** The productive labor force.
- **Elderly (65+ years):** The dependent group requiring healthcare and pensions.

Age Dependency Ratio (ADR)

The **Age Dependency Ratio (ADR)** is a measure used to assess the economic burden on the working-age population due to the non-working-age population. It indicates the proportion of a population that is dependent on the working-age group (usually considered to be ages 15-64) for economic support.

The **Age Dependency Ratio** can be calculated using the formula:

$$ADR = \frac{(P_{0-14} + P_{65+})}{P_{15-64}} \times 100$$

Where:

- P_{0-14} = Population of children (ages 0-14)
- P_{65+} = Population of elderly (ages 65 and above)
- P_{15-64} = Population of working-age adults (ages 15-64)

Interpreting the ADR:

- **Higher ADR**: Indicates a larger dependent population relative to the working-age population. This is typical in younger populations or aging societies, where a larger proportion is dependent (either children or elderly).
- **Lower ADR**: Indicates fewer dependents and a larger proportion of the population in the workforce. This often occurs in populations with lower birth rates and lower proportions of elderly individuals.

Types of Dependency Ratios

Child Dependency Ratio (CDR)

The **Child Dependency Ratio (CDR)** measures the number of children (0-14 years) for every 100 working-age individuals (15-64 years). It focuses on the economic burden imposed by children on the working-age population.

$$CDR = \frac{P_{0-14}}{P_{15-64}} \times 100$$

Where:

- P_{0-14} is the population aged 0-14 years (children).
- P_{15-64} is the working-age population (ages 15-64).

A higher CDR suggests a high proportion of children in the population, indicating higher dependency due to the need for schooling, healthcare, and other social services for the younger population.

Types of Dependency Ratios

Elderly Dependency Ratio (EDR)

The **Elderly Dependency Ratio (EDR)** measures the number of elderly individuals (65+ years) for every 100 working-age individuals (15-64 years). It focuses on the economic burden imposed by the elderly population on the working-age group.

$$EDR = \frac{P_{65+}}{P_{15-64}} \times 100$$

Where:

- P_{65+} is the elderly population (ages 65 and above).
- P_{15-64} is the working-age population (ages 15-64).

A higher EDR indicates an aging population and suggests that a greater portion of the population is dependent on pensions, healthcare, and other services.

Math

In Bangladesh, with a population of 45 million children (0–14), 100 million working-age individuals (15–64), and 7 million elderly people (65+), the government is assessing the pressure on the workforce caused by dependents. What are the dependency ratios in Bangladesh based on its population distribution, and how do these ratios impact the working-age population?

Solution:

In Bangladesh, the population distribution is as follows:

- Children (0–14): 45 million
- Working-age individuals (15–64): 100 million
- Elderly (65+): 7 million

Math

Child Dependency Ratio (CDR):

$$CDR = \frac{\text{Population aged 0–14}}{\text{Population aged 15–64}} \times 100 = \frac{45,000,000}{100,000,000} \times 100 = 45$$

Interpretation: For every 100 working-age individuals in Bangladesh, there are **45 children** who are dependent.

Elderly Dependency Ratio (EDR):

$$EDR = \frac{\text{Population aged 65+}}{\text{Population aged 15–64}} \times 100 = \frac{7,000,000}{100,000,000} \times 100 = 7$$

Interpretation: For every 100 working-age individuals, there are **7 elderly people** in Bangladesh.

Total Dependency Ratio (TDR):

$$TDR = \frac{\text{Population aged 0–14} + \text{Population aged 65+}}{\text{Population aged 15–64}} \times 100 = \frac{45,000,000 + 7,000,000}{100,000,000} \times 100 = 52$$

Interpretation: For every 100 working-age individuals, there are **52 dependents** (children and elderly). This indicates a moderate-to-high total dependency ratio.

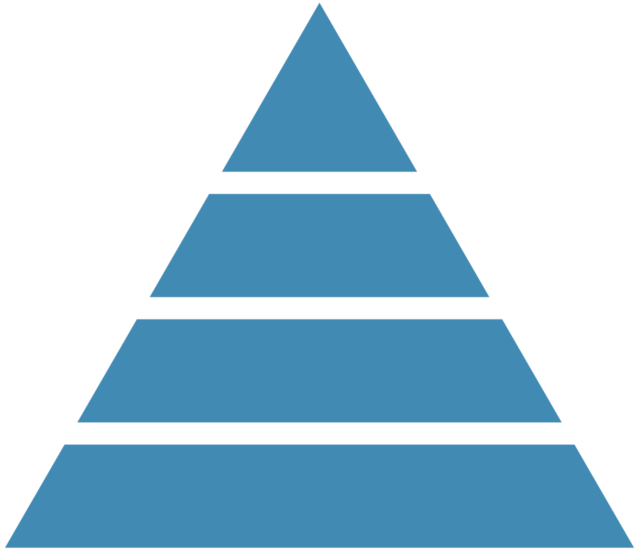
Age Distribution Graph / Population Pyramid

A population pyramid is a graphical representation that shows the distribution of various age groups in a population. It typically displays the number or percentage of males and females in various age groups, usually in 5-year cohorts.

- The x-axis represents the population size (or percentage).
- The y-axis shows age groups (typically from 0–4 to 85+).
- Males are generally shown on the left, and females on the right.

It is often shaped like **a pyramid** when the population has a **high birth rate and lower life expectancy** but may take other shapes depending on demographic trends.

Types of Population Pyramids



Different countries have unique population pyramids and for the same countries over the period of time shape changes. Population pyramids are typically classified into **three types**, depending on the demographic structure:

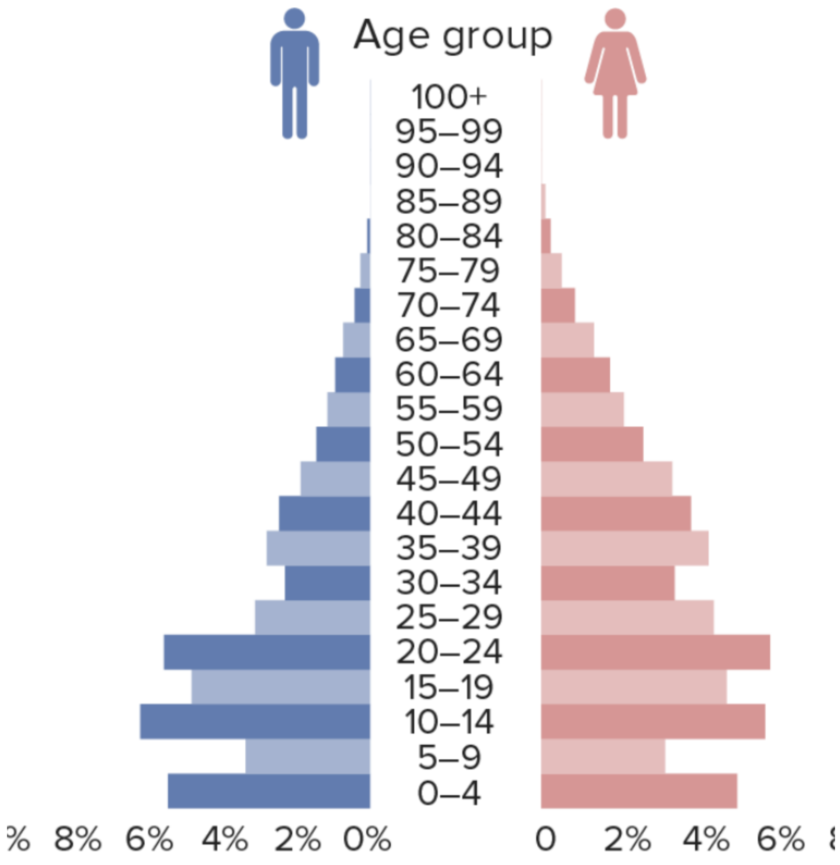
1. Expansive
2. Stationary
3. Constrictive

Expansive Pyramid:

- ✓ The population pyramid with broad base and with successive decline in the share of population of higher age groups is known as expansive pyramid.
- ✓ This pyramid represents situation of high fertility, high mortality, low life expectancy, higher population growth rates and low share of old age persons.

Key Features:

- ❑Wide base (large number of children and youth)
- ❑Narrow top (few elderly people)
- ❑Characteristic of developing or low-income countries



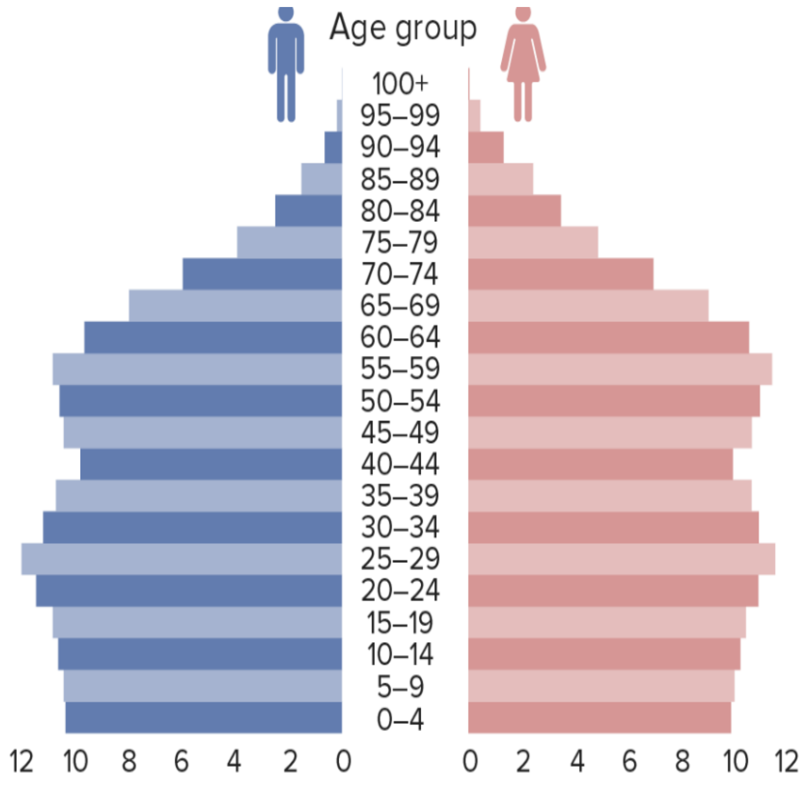
Stationary Pyramid

✓ A pyramid is described stationary when the share of population remains constant in different age groups over the period of time. It represents situation of low fertility, low mortality and high life expectancy.

✓ It indicates slow population growth or stable population. The stationary or near stationary population pyramid displays somewhat equal share of juvenile and adult age groups.

Key Features:

- ❑ Rectangular shape
- ❑ Nearly equal population in most age groups until old age
- ❑ Indicative of balanced growth and long life expectancy

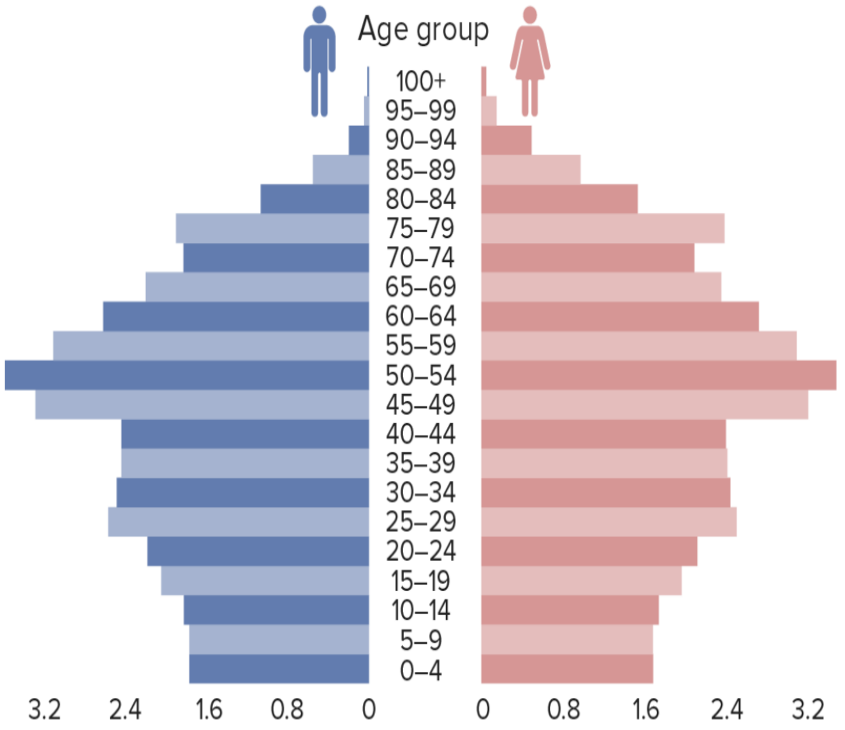


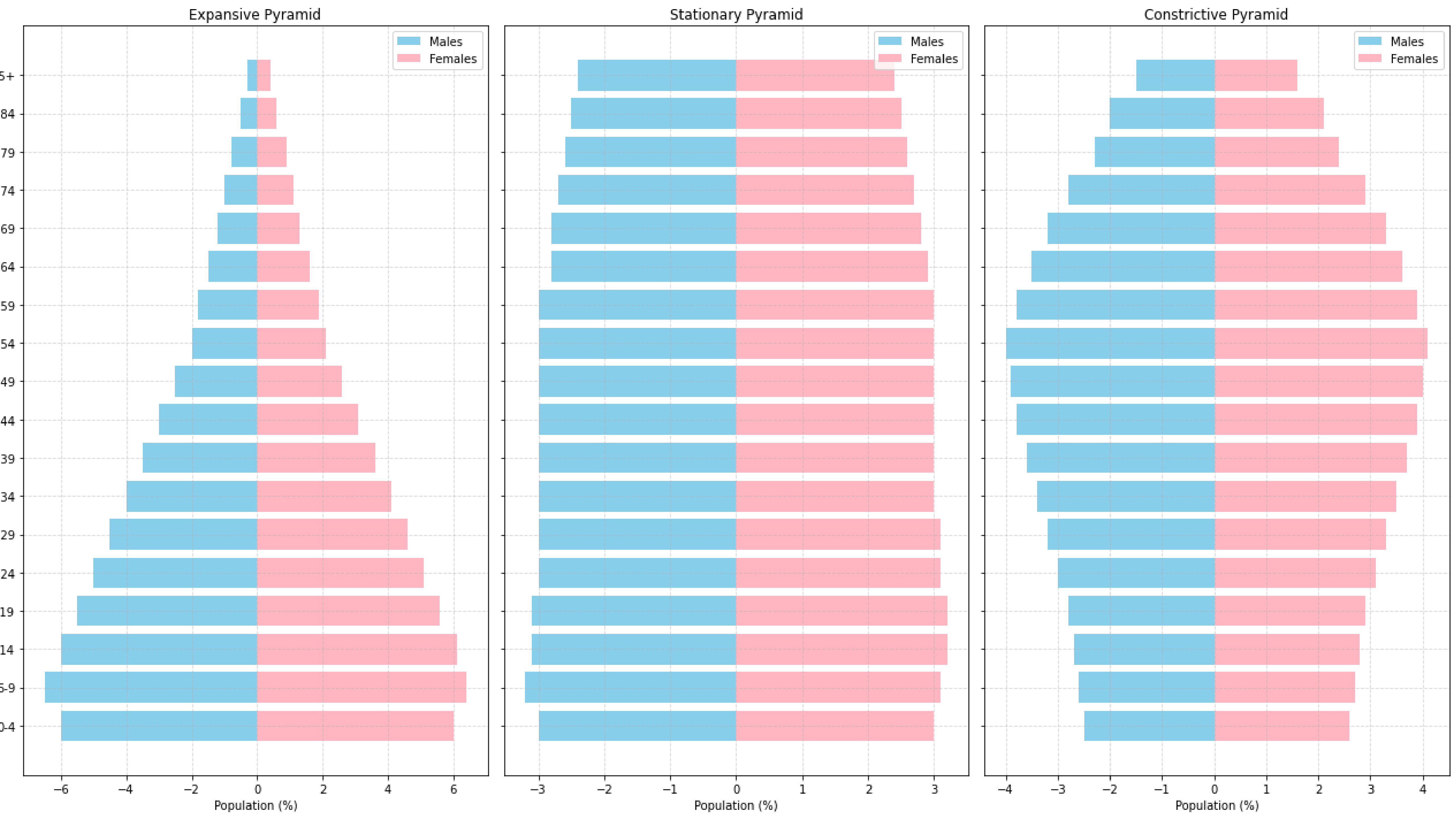
Constrictive Pyramid

- ✓ A constrictive pyramid has a narrower base than middle-age groups.
- ✓ It represents low fertility, low mortality, high life expectancy and ageing of population. It is typically associated with very advanced countries which have a high level of literacy, east access to birth control measures and very good health and medical facilities.

Key Features:

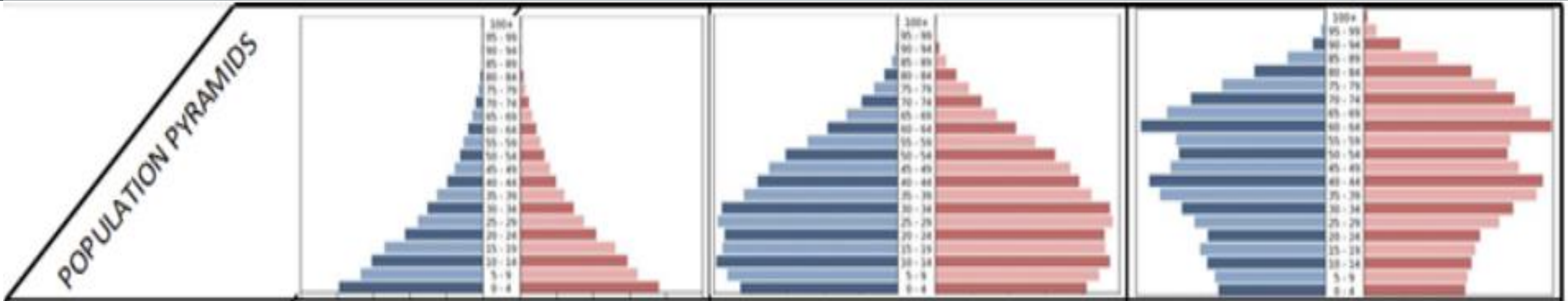
- ❑ Narrow base (fewer young people)
- ❑ Bulging middle (large number of working-age adults)
- ❑ Wider top compared to expansive pyramids (more elderly)





Expansive Pyramid
Stationary Pyramid
Constrictive Pyramid
Males
Females
Population (%)
5+
84
79
74
69
64
59
54
49
44
39
34
29
24
19
14
5-9
0-4
−6 −4 −2 0 2 4 6
−3 −2 −1 0 1 2 3
−4 −3 −2 −1 0 1 2 3 4

Types of Population Pyramid



TYPE OF PYRAMID	Expansive	Stationary	Contractive
DEVELOPMENT	Underdeveloped	Developing	Developed
Birth rate	Very high	Declining	Low
Death rate	Very high	High, but declining	Low
Life expectancy	Short	Increasing	Long
Age groups	Young - Adult - Elderly	Adult - Young - Elderly	Adult - Elderly - Young

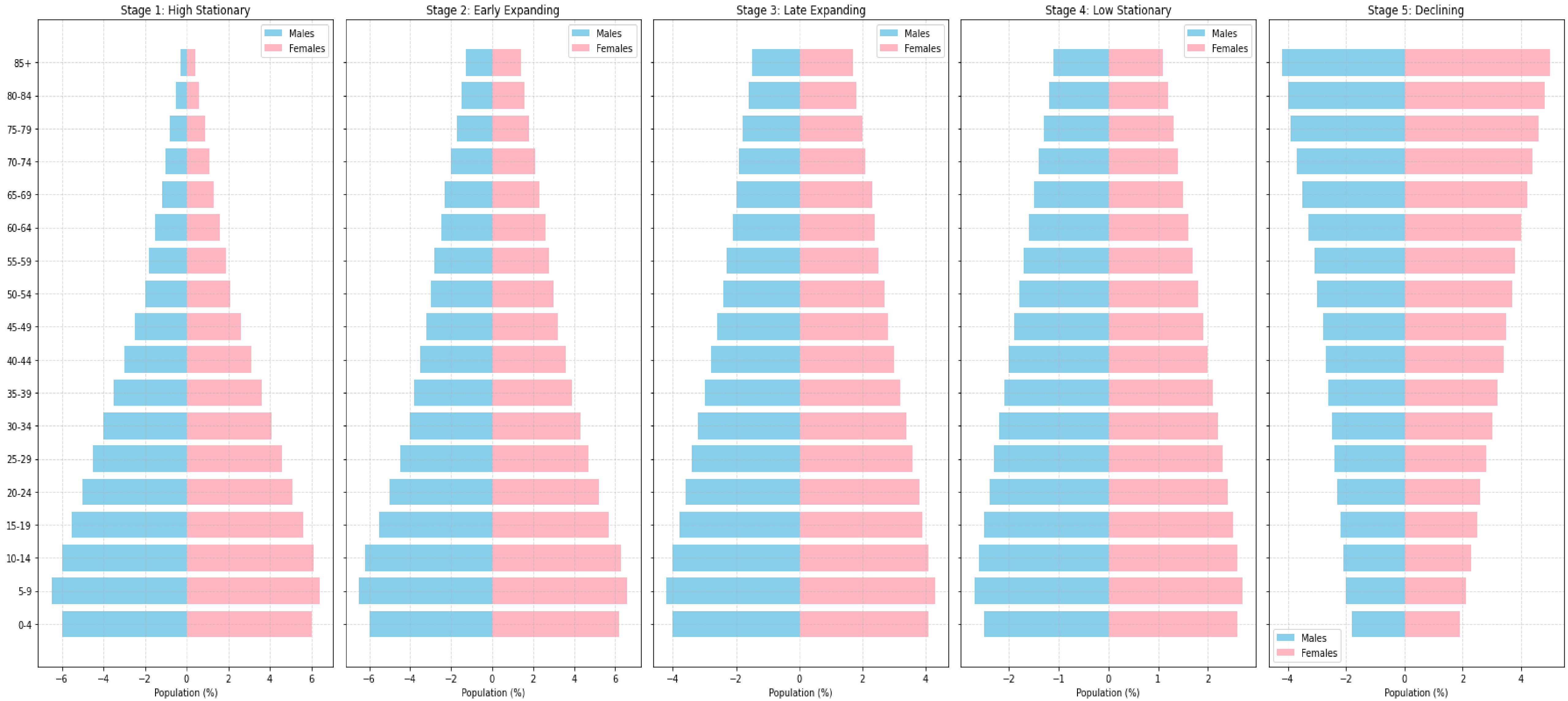
Population Pyramid and Demographic Transition

Demographic transition model (DTM) represents world population growth in past, present and future context on the basis of differential rates of fertility and mortality.

This model provides a general useful approximation of the changes which take place in populations of different countries over the period of time.

This model shows a particular pattern of demographic change from a high fertility and high mortality to a low fertility and low mortality.

Stages of the Demographic Transition Model



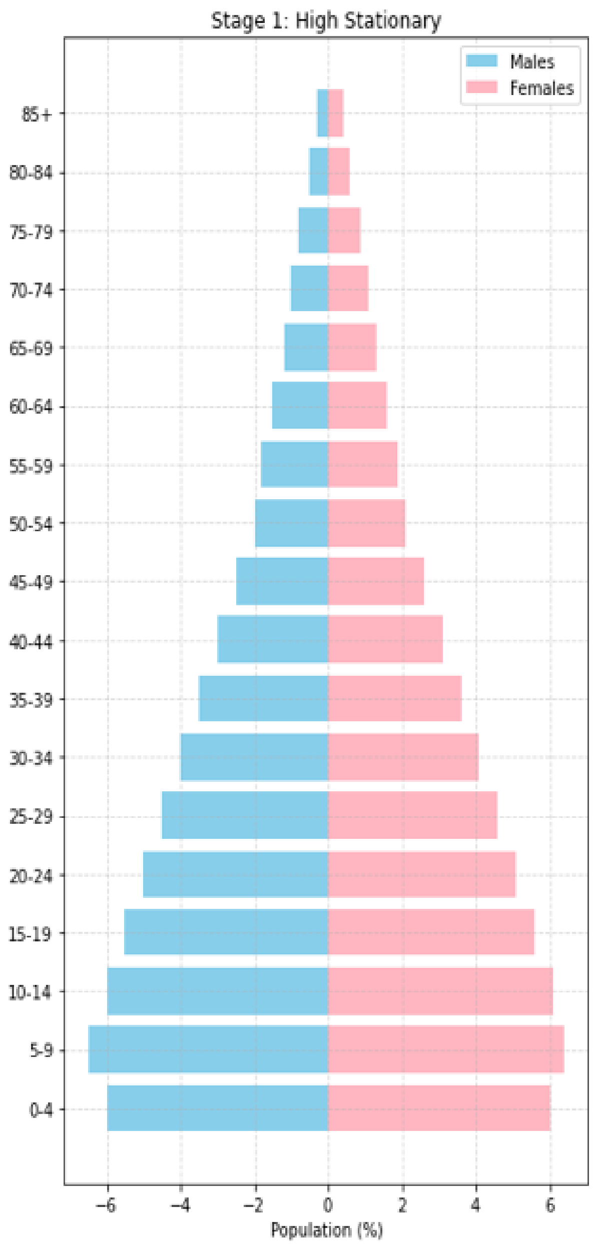
Stage 1: High Stationary

Characteristics:

- ✓ High birth rates and high death rates.
- ✓ The population remains stable or grows very slowly.
- ✓ Poor healthcare, sanitation, and nutrition lead to high infant mortality, diseases, and poor life expectancy.
- ✓ Life expectancy is low (typically under 40 years).

Population Pyramid:

- ✓ Broad base (many children) and narrow top (few elderly).
- ✓ High birth rates are offset by high death rates.



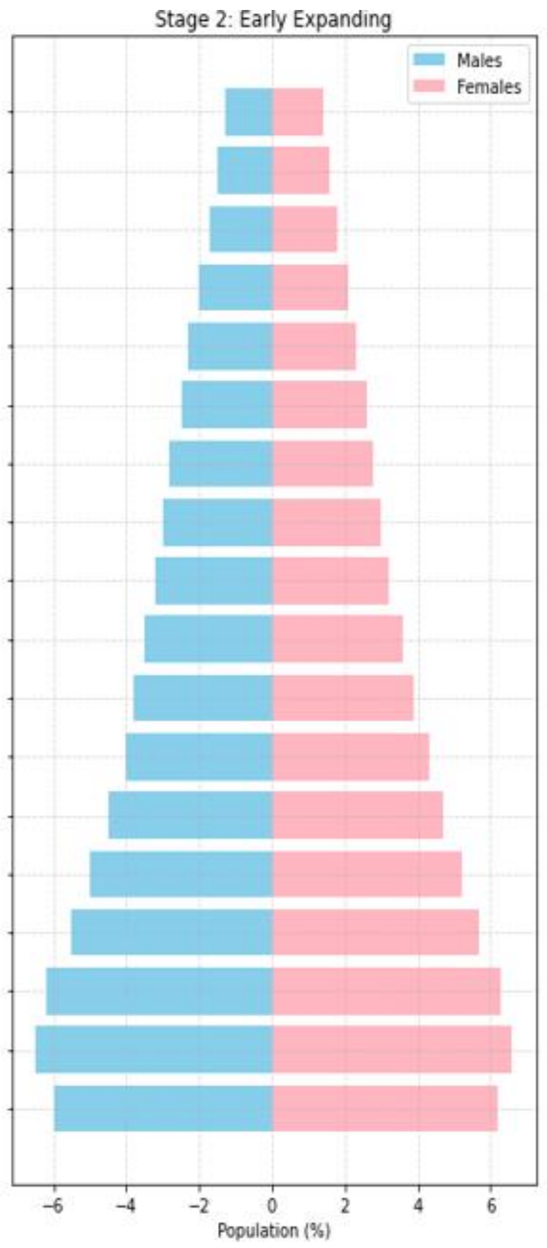
Stage 2: Early Expanding

Characteristics:

- ✓ High birth rates continue.
- ✓ Death rates begin to fall significantly due to improvements in healthcare, sanitation, and nutrition.
- ✓ Life expectancy starts to increase (typically 40-60 years).
- ✓ The population starts to grow rapidly as more children survive into adulthood.

Population Pyramid:

- ✓ Wide base (many children), narrow middle (fewer adults), and a narrow top (few elderly).
- ✓ The expansion of the base (young population) indicates rapid population growth



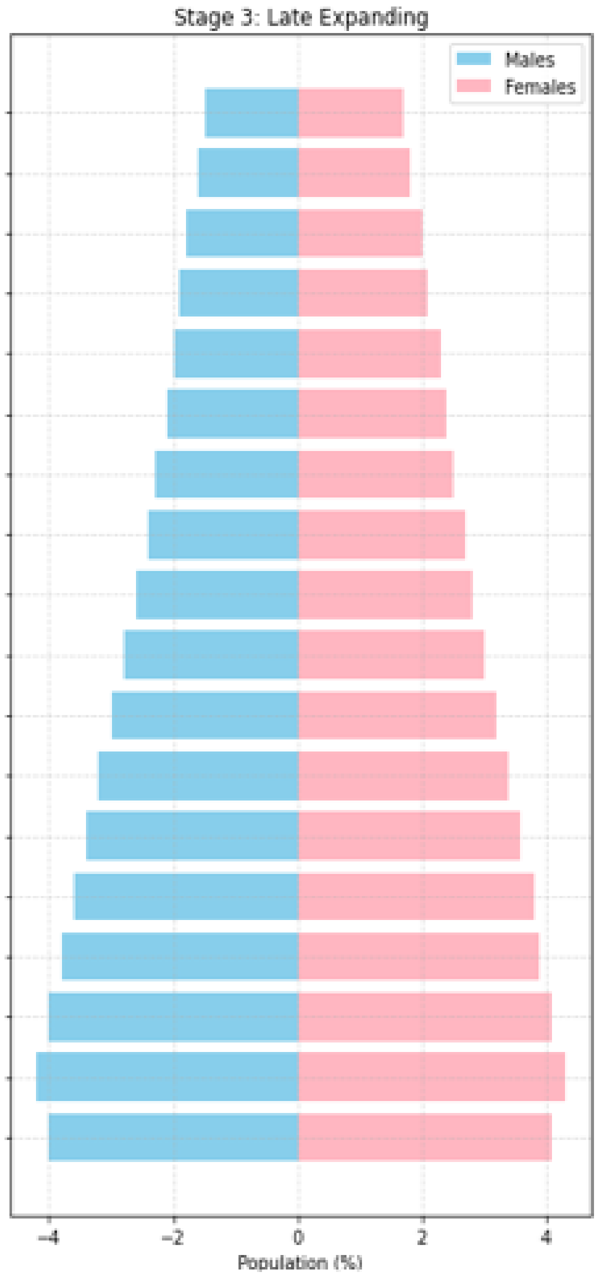
Stage 3: Late Expanding

Characteristics:

- ✓ Birth rates begin to decline as family planning, improved standards of living, and access to education become more widespread.
- ✓ Death rates continue to fall, but at a slower rate.
- ✓ Life expectancy continues to rise (typically 60-75 years).
- ✓ Population growth begins to slow down, and the country begins to stabilize.

Population Pyramid:

- ✓ Narrowing base (fewer children), wider middle (more working-age people), and a wider top (more elderly people).
- ✓ The pyramid flattens, and the growth rate declines.



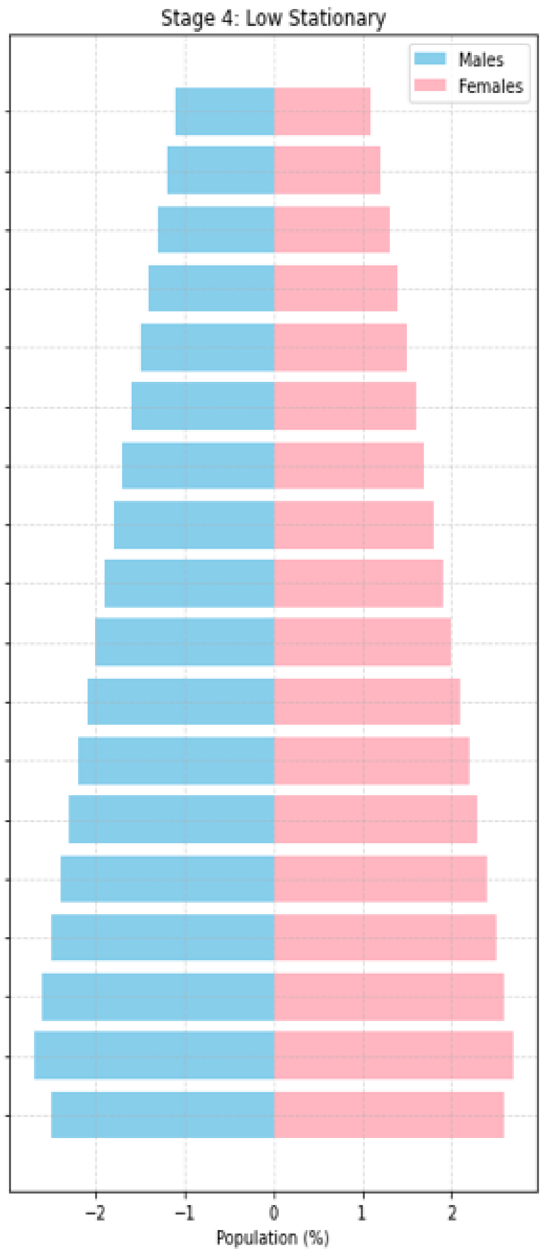
Stage 4: Low Stationary

Characteristics:

- ✓ Both birth rates and death rates are low.
- ✓ Population growth is stable or very slow, with some countries possibly experiencing negative growth if birth rates fall below death rates.
- ✓ Life expectancy is high (75+ years), and healthcare systems are well-developed.
- ✓ Family planning is widely practiced, and fertility rates are low.

Population Pyramid:

- ✓ Narrow base (few children), wide middle (large working-age population), and wide top (more elderly).
- ✓ Population growth stabilizes, but an aging population becomes a concern.



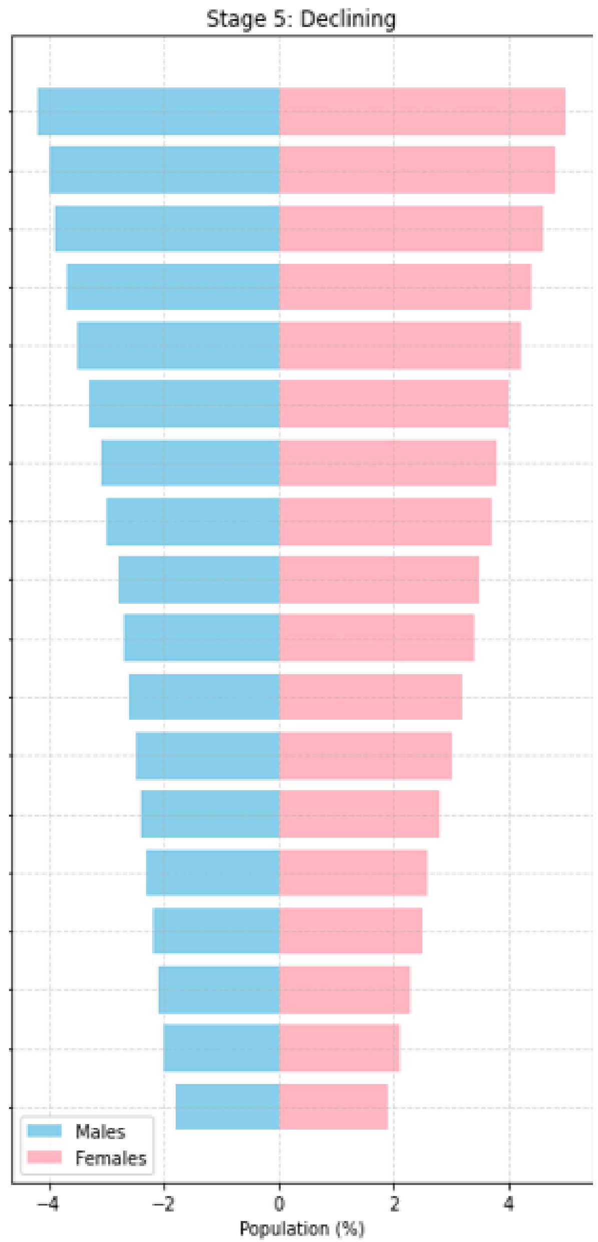
Stage 5: Declining

Characteristics:

- ✓ Very low birth rates (below death rates).
- ✓ Death rates remain low but might slightly increase due to an aging population.
- ✓ Population decline begins as the number of births falls below the number of deaths.
- ✓ High life expectancy continues, but the country faces challenges related to an aging population.
- ✓ Economic challenges emerge, such as fewer workers to support the elderly.

Population Pyramid:

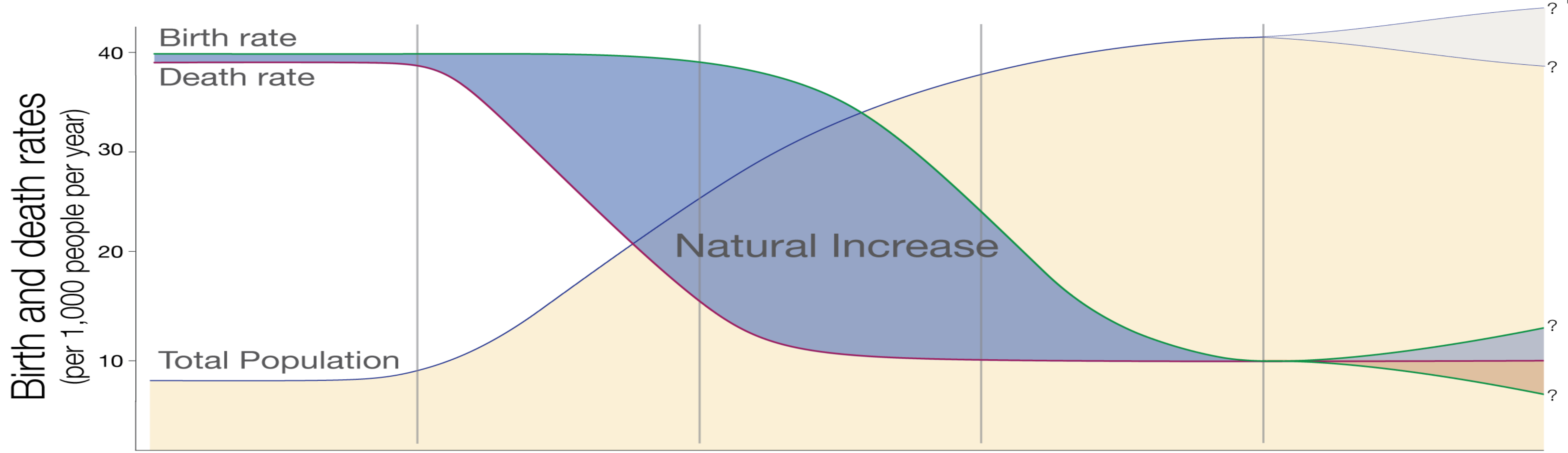
- ✓ Very narrow base (few children), wide middle, and a very broad top (many elderly).
- ✓ The pyramid is **inverted**, indicating a shrinking population.



The five stages of the demographic transition

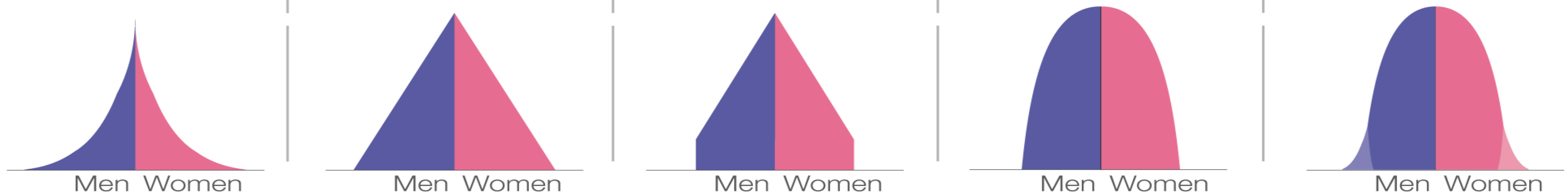
The demographic transition is a model that describes why rapid population growth is a temporary phenomenon.

Our World in Data



	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
Birth rate	High	High	Falling	Low	Yet to be seen (Possibly falling further, possibly rising again)
Death rate	High	Falls rapidly	Falls more slowly	Low	Low
Natural increase	Stable or slow increase	Rapid increase	Increase slows down	Falling and then stable	Little change

Population Pyramid



Sex composition: Sex Ratio

The sex ratio is a demographic indicator that refers to the ratio of males to females in a population.

It provides valuable insights into the gender composition of a population at different life stages, including birth, adulthood, and overall population structure.

Primary Sex Ratio/ Sex Ratio at Birth (SRB)

- ✓ The primary sex ratio refers to **the ratio of male to female births.**
- ✓ The primary sex ratio generally tends to favor males slightly. In most populations, it is about 105 males for every 100 females.

$$SR_P = \frac{\text{Number of male births}}{\text{Number of female births}} \times 100$$

Example: If the number of male births is 2616 and female births are 2464, the primary sex ratio can be calculated as:

$$SR_P = \frac{2616}{2464} \times 100 = 106\%$$

This means that for every 100 females born, approximately **106 males** are born.

Secondary Sex Ratio

- ✓ The secondary sex ratio refers to the ratio of males to females in the total population **(including all age groups).**
- ✓ This includes both natural population sex imbalances at birth and changes due to differential mortality or migration.

$$SR_S = \frac{\text{Total number of males in the population}}{\text{Total number of females in the population}} \times 100$$

Example: If the total number of males in a population is 7210976 and total females is 7210976:

$$SR_S = \frac{7210976}{7210976} \times 100 = 100$$

This indicates a **balanced** population in terms of males and females.

Tertiary Sex Ratio

The tertiary sex ratio refers to the ratio of males to females in the **working-age population or in other specific age** groups that are relevant to the economy or labor force.

$$SR_T = \frac{\text{Number of males in working age group}}{\text{Number of females in working age group}} \times 100$$

Example: If there are 300,000 males and 290,000 females in the working-age group, the tertiary sex ratio would be:

$$SR_T = \frac{300000}{290000} \times 100 = 103.45$$

This indicates there are more working-age males than females.

Factors Influencing Sex Ratios

- ✓ Genetics and environmental factors during conception can influence the primary sex ratio.
- ✓ In some societies, son preference may influence family planning and lead to higher birth rates of males through sex-selective practices.
- ✓ The availability of healthcare services, particularly maternal and infant care, can influence infant mortality rates, impacting the secondary and tertiary sex ratios.
- ✓ International and internal migration can significantly affect the sex ratio in specific regions, as certain areas might attract more male laborers.

Measurement of Age and Digit Preference (Age Heaping)

- ✓ Age Heaping refers to the tendency of people to report their age as a rounded number (e.g., 20, 30, 40, 50), instead of their exact age.
- ✓ Often observed in populations where birth registration is incomplete or inaccurate, and individuals may estimate their age or round it off to the nearest multiple of 5 or 10.
- ✓ Age heaping is particularly common in regions with low literacy rates, poor documentation systems, and limited access to accurate age verification methods.

To quantify and evaluate age heaping, there are several statistical methods and indices available, with **Myer's Index** and **Whipple's Index** being two of the most commonly used.

Whipple's Index (Terminal Digit 0 or 5)

Whipple’s Index is used to assess age heaping. It focuses on counting the number of people whose ages end in digits 0 or 5. Whipple’s Index is simpler than Myer’s Index and is used primarily for evaluating age data in developing countries where there is likely to be more age misreporting.

Steps:

- ✓ The index score is obtained by summing the number of persons in the age range 23 and 62 inclusive, who report ages ending in 0 and 5, dividing that sum by the total population between ages 23 and 62 years inclusive, and multiplying the result by 5.
- ✓ Restated as a percentage, index scores range between 100 (no preference for ages ending in 0 and 5) and 500 (all people reporting ages ending in 0 and 5)

$$\frac{\sum (P_{25} + P_{30} + \ldots P_{55} + P_{60})}{1/5 \sum (P_{23} + P_{24} + P_{25} + \ldots P_{60} + P_{61} + P_{62})} \times 100$$

$$= \frac{\sum_{23}^{62} P_a \text{ ending in 0 or 5}}{1/5 \sum_{23}^{62} P_a} \times 100$$

$P_{25}, P_{30}, \ldots, P_{60}$ = Populations in age groups that end in 0 or 5.

$P_{23}, P_{24}, \ldots, P_{62}$ = Populations in all other age groups (non-rounded ages).

The **denominator** is normalized by dividing by 1/5 to adjust for the population's total age groups.

Whipple's Index (Terminal Digit 0)

If the heaping is specifically on the terminal digit "0", the formula for **Whipple's Index** is:

$$\text{Index} = \frac{P_{30} + P_{40} + P_{50} + P_{60}}{\left(\frac{1}{10}\right)\sum_{r=23}^{62} P_r} \times 100$$

Where,

$P_{30}, P_{40}, P_{50}, P_{60}$ represent the populations in age groups where ages end in 0 (rounded ages).

$\sum P_r$ is the sum of the population in all the other age groups, from age groups ending in digits 23-62.

The denominator normalizes the result by dividing by $\frac{1}{10}$, adjusting for the expected distribution of terminal digits.

The UN recommended standard for measuring the age heaping using Whipple's Index

Whipple's index	Quality of data	Deviation from perfect
< 105	Highly accurate	< 5%
105–109.9	Fairly accurate	5–9.99%
110–124.9	Approximate	10–24.99%
125–174.9	Rough	25–74.99%
> 175	Very rough	≥ 75%

Cohort Analysis

Cohort analysis is a powerful research method used in **demography** to study groups of individuals (known as **cohorts**) who share common characteristics.

These characteristics typically include being born in the same year, or experiencing a significant event during the same time period.

Cohorts can be analyzed to uncover how different generations experience social, economic, and demographic changes over time.

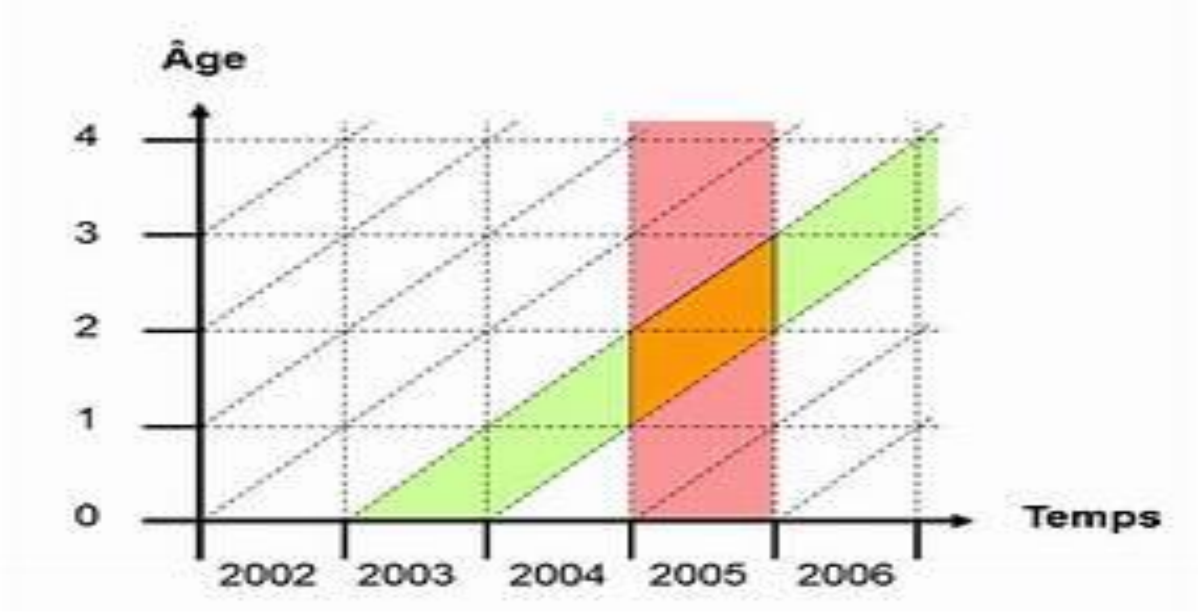
Lexis diagram

A graphic technique developed apparently independently by several people, but largely attributed to Wilhelm Lexis (hence, the name “Lexis diagram”), that is designed to reveal the relationship between age, time, and population change, with particular applications to cohort analysis, life table construction, and population estimation.

Lexis Diagram

The **Lexis diagram** is a **graphical tool** that helps visualize the relationship between three key demographic variables:

1. **Age**
2. **Time (or Period)**
3. **Cohort**



Structure of the Lexis Diagram:

Age Axis (Y-axis):

This vertical axis represents the age of individuals. As you move upward, individuals grow older. Each point along this axis corresponds to the age of a specific individual or group of individuals.

Time (Period) Axis (X-axis):

This horizontal axis represents calendar time or periods. As you move rightward, time progresses. The timeline could represent any period (such as years or decades).

Cohort Diagonal:

The diagonal lines on the Lexis diagram represent cohorts (groups of people born in the same year). Each diagonal corresponds to a specific cohort and reflects how individuals in that cohort age as time passes. The slope of the diagonal indicates the speed at which a cohort ages.

Components of the Lexis Diagram

- **Cohort Effect**:

A **cohort effect** refers to the variations in behaviors or characteristics that are unique to individuals born in the same year (or generation). It is represented by the diagonal lines in the diagram. Each cohort's experiences and outcomes are influenced by the time and environment they live in.

- **Period Effect**:

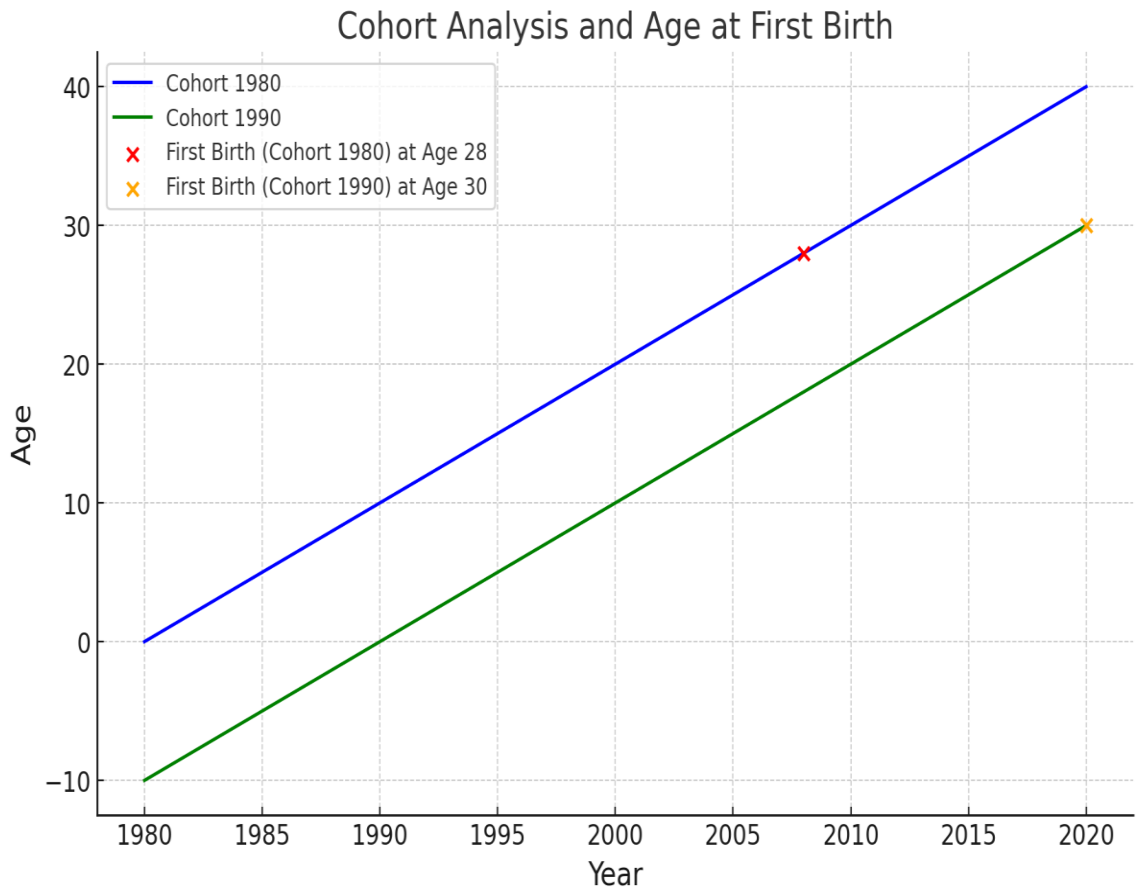
A **period effect** refers to the changes that affect all individuals in a population at the same time, regardless of their age or cohort. These effects are represented along the horizontal axis (the time axis). For instance, a pandemic like COVID-19, or a major social or economic change that occurs at a specific point in time, will influence all individuals, regardless of their age or cohort.

- **Age Effect**:

An **age effect** refers to the changes that occur as individuals grow older. This can include biological, psychological, and social changes. The **age effect** is reflected by the upward movement along the vertical axis (the age axis) of the Lexis diagram.

Example of Lexis Diagram

- The cohort of women born in 1980 will have a diagonal line that starts at age 0 in 1980. As time progresses, these women will age and eventually reach 40 years old in 2020.
- Cohort 1990: The cohort of women born in 1990 will have a similar diagonal line, but it will begin at 1990 and move upward as the women in this group age. By 2020, the women in this cohort will be 30 years old.
- Event of Interest (First Birth): Now, if we are interested in studying the age at first birth, we can track this event on the Lexis diagram.
- For example, if women born in 1980 tend to have their first child at an average age of 28 and women born in 1990 tend to have their first child at an average age of 30.



Reproduction Rates

Reproduction rates are used to understand how a population reproduces and the extent to which it can replace itself across generations.

The Gross Reproduction Rate (GRR) and the Net Reproduction Rate (NRR) are two critical indicators that help measure fertility and replacement potential within a population.

Gross Reproduction Rate (GRR)

The Gross Reproduction Rate (GRR) is a special case of the Total Fertility Rate (TFR). Unlike TFR, which calculates the total number of children born to a woman, GRR focuses specifically on the number of daughters.

The formula for **GRR** is:

$$\text{GRR} = \sum_{x=w_1}^{w_2} \left(\frac{B_x}{P_x}\right) \times \left(\frac{B_f}{B_t}\right) \times k$$

Where:

- B_x = Number of live births to mothers of age x,
- P_x = Midyear female population of age x,
- B_f/B_t = The **proportion of births that are female** (i.e., **sex ratio at birth**),
- k = A constant (typically 1, 100, or 1000) depending on the unit used for the calculation.

Gross Reproduction Rate (GRR)

Interpretation of Gross Reproduction Rate (GRR)

- **GRR = 1**: This means the cohort of women will have exactly one daughter per woman, indicating that the population is replacing itself perfectly.
- **GRR > 1**: This indicates **population growth**, where each cohort of women is having more daughters than needed for replacement.
- **GRR < 1**: This indicates **population decline**, as the cohort is having fewer daughters than necessary to replace itself.

If data for **female births** are available directly, the formula becomes:

$$\text{GRR} = \sum_{x=w_1}^{w_2} \left(\frac{B_{f,x}}{P_x} \right) \times k$$

Where:

- $B_{f,x}$ = Number of **female births** to mothers of age x,
- P_x = Midyear female population of age x.

For 5-Year Age Groups:

When fertility data is grouped into **5-year age intervals**, the formula becomes:

$$\text{GRR} = 5 \sum_{i=1}^{7} \left(\frac{B_i}{P_i} \right) \times \left(\frac{B_f}{B_t} \right) \times k$$

Where:

- B_i = Number of **female births** in the i^{th} age group (e.g., 15–19, 20–24, etc.),
- P_i = Midyear female population in the i^{th} age group,
- B_f / B_t = Proportion of female births.

Net Reproduction Rate (NRR)

The Net Reproduction Rate (NRR) adjusts the Gross Reproduction Rate (GRR) by considering mortality.

While the GRR assumes no mortality and calculates the potential number of daughters in an idealized scenario, the NRR incorporates the reality that some females will not survive to the end of their childbearing years.

Formula for Net Reproduction Rate (NRR):

The formula for **NRR** is:

$$\mathrm{NRR} = \sum_{x=w_1}^{w_2} \left(\frac{B_x}{P_x}\right) \times \left(\frac{L_x}{L_0}\right) \times \left(\frac{B_f}{B_t}\right)$$

Where:

- $B_x/P_x = f_x$ is the **age-specific fertility rate (ASFR)** at age x,
- L_x/L_0 is the **life-table survival rate** at age x,
- B_f/B_t is the **proportion of female births** (sex ratio at birth),
- L_0 is the **radix of the life table** (often set to 100,000).

Net Reproduction Rate (NRR)

Interpretation of Net Reproduction Rate (NRR):

- **NRR = 1**: The population is exactly replacing itself, meaning the number of daughters is sufficient to replace the female population.
- **NRR > 1**: The population is **growing**, as the cohort of women is having more daughters than necessary for replacement, even when accounting for mortality.
- **NRR < 1**: The population is **declining**, as the cohort is not having enough daughters to replace themselves, factoring in mortality.

When using **5-year age groups**, the formula for **NRR** becomes:

$$NRR = 5\frac{B^f}{B^t}\sum_{i=1}^{7}\frac{B_i}{P_i}\cdot\frac{{}_5L_x}{{}_5l_o} = \frac{B^f}{B^t}\sum_{i=1}^{7}\frac{B_i}{P_i}\cdot\frac{{}_5L_x}{100{,}000}$$

Where:

- B_i = Number of **female births** in the i^{th} age group,
- P_i = Midyear female population in the i^{th} age group,
- L_x/L_0 = Life-table survival rate for the i^{th} age group.

GRR and NRR calculation

- *Step 1.* Compute the age-specific birthrate per female, and enter it in column 1.
- *Step 2.* From an appropriate life table, find the number of years lived by females in the stationary population for each interval of age. In an abridged life table, 5*Lx* will be given directly. (In a life table with single years of age, 5*Lx* can be obtained by summing five successive values of *Lx* or by subtracting *Tx*+5 from *T*5.)
- *Step 3.* Divide 5*Lx* by 100,000, and enter the result in column 2.
- *Step 4.* Multiply each entry of column 2 by the corresponding entry of column 1 and enter the result in column 3.
- *Step 5.* To obtain the gross reproduction rate, multiply the sum of the entries in column 1 by the proportion of births that are female (.48551) and then by the factor *5.*
- *Step 6.* To obtain the net reproduction rate, apply the factor .48551 to the sum of column 3.

Age of mother (years)	Age-specific birth rate per female ${}_5f_x^T$ (1)	Survival rate[1] $\frac{{}_5L_x^F}{\ell_0^F}$ (2)	Number of births ${}_5f_x^T \cdot \frac{{}_5L_x^F}{\ell_0^F}$ (1) × (2) = (3)
15–19	0.03301	4.94560	0.16327
20–24	0.08031	4.93198	0.39610
25–29	0.11825	4.91951	0.58174
30–34	0.10404	4.90505	0.51031
35–39	0.04403	4.88659	0.21516
40–44	0.00790	4.86117	0.03842
45–49	0.00041	4.81951	0.00198
Sum	0.38796	X	1.90698

$$GRR = 5\frac{B^F}{B^T}\sum {}_5f_x^T = 5(.48551)(0.38796) = 0.94179$$

$$NRR, \text{ or } R_0 = 5\frac{B^F}{B^T}\sum {}_5f_x^T \cdot \frac{{}_5L_x^F}{5 \cdot \ell_0^F}$$

$$= \frac{B^F}{B^T} \cdot \sum {}_5f_x^T \cdot \frac{{}_5L_x^F}{\ell_0^F}$$

$$= (.48551)(1.90698) = 0.92586$$

Stable Population Model

- The Stable Population Model is a theoretical framework in demography used to study populations with constant fertility and mortality rates over time.
- The assumption is that a population’s age-specific birthrates and death rates remain unchanged, and there is no migration.
- This model is crucial for understanding the long-term dynamics of populations and their age structures.

Stable Population Model

- **Constant Age-Specific Birth and Death Rates**: The birth and death rates for each age group remain fixed over time.
- **No Migration**: The population is assumed to be closed, meaning there is no movement of individuals into or out of the population.
- **Stable Age Distribution**: Over time, the population's age composition reaches a stable distribution.

In 1925, **Lotka** and **Dublin** proved that such a closed population would eventually have a **constant rate of natural increase**, which is the rate at which the population grows or shrinks over time.

Stationary Population

A stationary population is a special case of a stable population, and although no real population is ever perfectly stationary, the concept provides a **valuable benchmark** for understanding population dynamics.

- The term “stationary” in this context does not merely mean “unchanging,” but more precisely refers to a population in which **births exactly balance deaths**, leading to **zero population growth**. This implies that the **total size of the population remains constant over time**, and more interestingly, the **age distribution also remains unchanged**.

Assumptions

- **Fertility and mortality rates are constant over time.**
- **No migration.**
- **Population size remains constant.**
- **Birth rate equals death rate:** $b = d$
- The **survivorship function** $l(x)$ describes how many individuals are alive at age x out of an original cohort l_0.
- $p(x) = \frac{l(x)}{l_0}$: the probability of surviving from birth to age x.
- T_0: the total number of **person-years** lived by this cohort from birth to death.
- e_0: **life expectancy at birth**, i.e., the average number of years a newborn is expected to live.

$$e_0 = \frac{T_0}{l_0}$$

From Life Table to Age Distribution

In a stationary population, the **age distribution** $c(x)$ is given by:

$$c(x) = b \cdot p(x) \tag{12.60}$$

This expresses that the proportion of people aged x is the product of:

- the birth rate b, and
- the probability of surviving to age x.

To ensure that this is a proper distribution (i.e., the total population sums to 1), integrate over the entire lifespan ω:

$$\int_0^\omega c(x)dx = \int_0^\omega b \cdot p(x)dx = b \int_0^\omega p(x)dx = 1$$

Solving for b:

$$b = \frac{1}{\int_0^\omega p(x)dx} \tag{12.61}$$

From Life Table to Age Distribution

From life table theory, the **total person-years lived** by the birth cohort is:

$$T_0 = \int_0^{\omega} l(x)dx$$

Dividing by l_0 gives the **life expectancy at birth**:

$$e_0 = \frac{T_0}{l_0} = \frac{\int_0^{\omega} l(x)dx}{l_0}$$

Now, substitute $p(x) = \frac{l(x)}{l_0}$ into the earlier expression:

$$\int_0^{\omega} p(x)dx = \int_0^{\omega} \frac{l(x)}{l_0}dx = \frac{1}{l_0}\int_0^{\omega} l(x)dx = \frac{T_0}{l_0} = e_0$$

Thus:

$$b = \frac{1}{e_0} \qquad (12.62)$$

Interpretation

- The number of births per year is equal to the number of deaths per year.
- These are each equal to the reciprocal of the average number of years each person lives.
- If life expectancy increases, the population becomes older and the birth (and death) rate declines to maintain population size.
- Conversely, if people have shorter lifespans, the birth rate must be higher to keep the population stationary.

Quasi-Stable Population

A quasi-stable population is a population on its way to becoming stable but not fully there yet. It happens when fertility and mortality rates have become constant or nearly constant, but the age structure hasn’t settled into the stable pattern.

- It usually arises after recent changes in mortality or fertility rates.
- The population grows exponentially at a constant rate r, but its age distribution is still adjusting.
- The population’s growth rate r is constant, but the age structure changes over time—unlike a stable population where both are constant.

Quasi-Stable Population

The concept is particularly useful for populations experiencing demographic transition, such as those moving from high fertility and high mortality to low fertility and low mortality. During such a transition, even if fertility and mortality stabilize at new levels, the legacy of earlier patterns keeps the age distribution from settling immediately.

Mathematically, a quasi-stable population has an **exponential growth rate** r (constant over time), but unlike a stable population, the **function** $c(x, t)$—which describes the proportion of the population at age x at time t—**depends on time**. Over time, however:

$$\lim_{t \to \infty} c(x, t) = c(x)$$

where $c(x)$ is the age structure of the stable population that would result if current fertility and mortality rates continued indefinitely.

Why Do Quasi-Stable Populations Occur?

Demographic Transitions

Populations do not instantly become stable after changes in fertility or mortality.

- When mortality declines (people live longer), the population starts to grow faster.
- If fertility remains constant, the population grows exponentially.
- However, the age distribution is skewed by past mortality patterns and does not immediately match the stable age distribution predicted by current rates.
- The lag time in the age structure adjusting causes quasi-stability.

Impact of Changing Mortality

- Mortality decline means more people survive to older ages, causing the population to **age**.
- This changes the proportion of different age groups over time.
- Until the population reaches a new equilibrium, the age distribution is **not fixed**, making it quasi-stable.

Mathematical Adjustments for Quasi-Stability Model

The foundational work by **Coale and Demeny (1966)** introduced a method to approximate the effect of declining mortality by applying an adjustment to the birth and death rates derived from stable population theory.

They developed an **index of mortality change**, denoted I, which is a function of how recently and how significantly mortality has declined:

$$I = 17.8 \left(\frac{r_0 - r}{t} \right)$$

- r_0: Intrinsic rate of natural increase before the mortality decline.
- r: Current intrinsic rate under new mortality.
- t: Time elapsed (in years) since the onset of mortality decline.

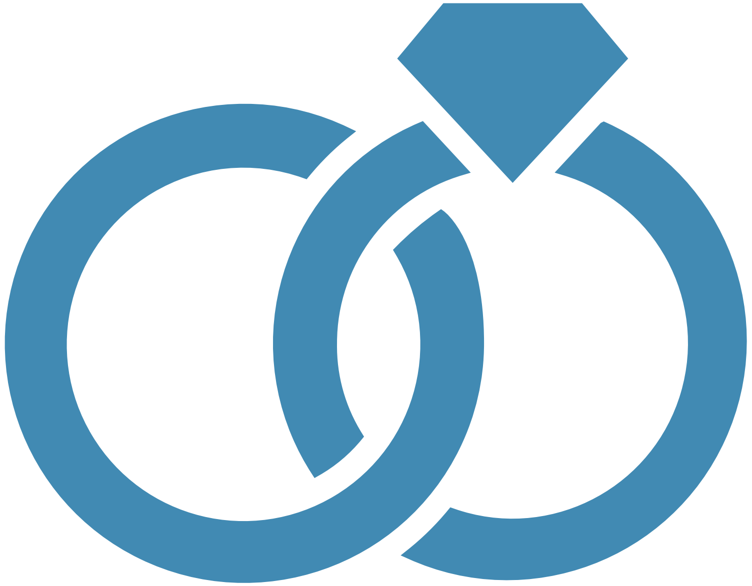
This index I adjusts the stable population assumptions to better fit populations where mortality has recently fallen but the age distribution hasn't caught up yet. Higher values of III correspond to more recent or more rapid mortality decline, leading to more significant deviation from stability.

Examples

- Suppose a country has undergone rapid improvements in public health, reducing infant and adult mortality rates dramatically over the last two decades. Fertility, however, has remained largely unchanged. The result is a sudden increase in the number of people living into older age groups.
- Initially, the population structure would still show a **bulge in the younger age groups** (due to historically high birth rates) and **increasing proportions in middle and older age groups** (due to falling mortality). This mix of new and old demographic characteristics means the population does not fit the theoretical stable model—it is quasi-stable.

Population structure

Feature	Stable Population	Stationary Population	Quasi-Stable Population
Growth Rate r	Constant, can be positive or negative	Exactly zero (r=0)	Constant but age structure still changing
Age Distribution	Constant over time	Constant over time	Changes over time, gradually approaching stable
Population Size	Grows or declines exponentially	Constant	Grows or declines exponentially
Fertility & Mortality	Constant	Constant	Constant
Migration	None	None	None
Applications	Long-term demographic projections	Baseline model for demographic equilibrium	Medium-term projections during demographic changes



Nuptiality

Nuptiality refers to the study of marriage patterns within a population. It encompasses aspects like the age at marriage, timing of marriage, and marriage rates across different segments of the population.

Understanding nuptiality is crucial for predicting demographic changes, such as population growth, fertility patterns, and the socio-economic status of various groups.

In countries like Bangladesh, nuptiality has direct implications for issues like gender equity, education, and family planning.

Monogamy

Monogamy refers to a marriage system where one person is married to only one spouse at a time. This is the most widely practiced form of marriage worldwide, especially in Western cultures and in countries where civil or religious laws require such an arrangement.

Key Characteristics

One-to-one relationship between spouses.	Legal and social recognition of the exclusive union between two individuals.	Most common in modern societies.

Types of Marriage

1. Monogamy
2. Polygamy
3. Polyamory

Polygamy

Polygamy involves one person having multiple spouses simultaneously.

There are two types of polygamy: **polygyny** and **polyandry**.

Polygyny

A form of polygamy in which one man has multiple wives simultaneously.

Prevalence: Polygyny is more common in various **African**, **Middle Eastern**, and **Southeast Asian** societies, particularly in **Muslim** communities where it is permitted under certain conditions by Islamic law.

Key Characteristics:

- A single man marries multiple women.
- Women often share their husband’s resources, but may have different social roles.

Polyandry

A form of polygamy in which one woman has multiple husbands simultaneously.

Prevalence: Polyandry is rare and typically practiced in specific regions such as **Tibet**, **Nepal**, and some parts of **India**.

Key Characteristics:

- One woman is married to several men, often brothers.
- Polyandry is often linked to **economic reasons**, such as limiting population growth in societies with limited resources or land.

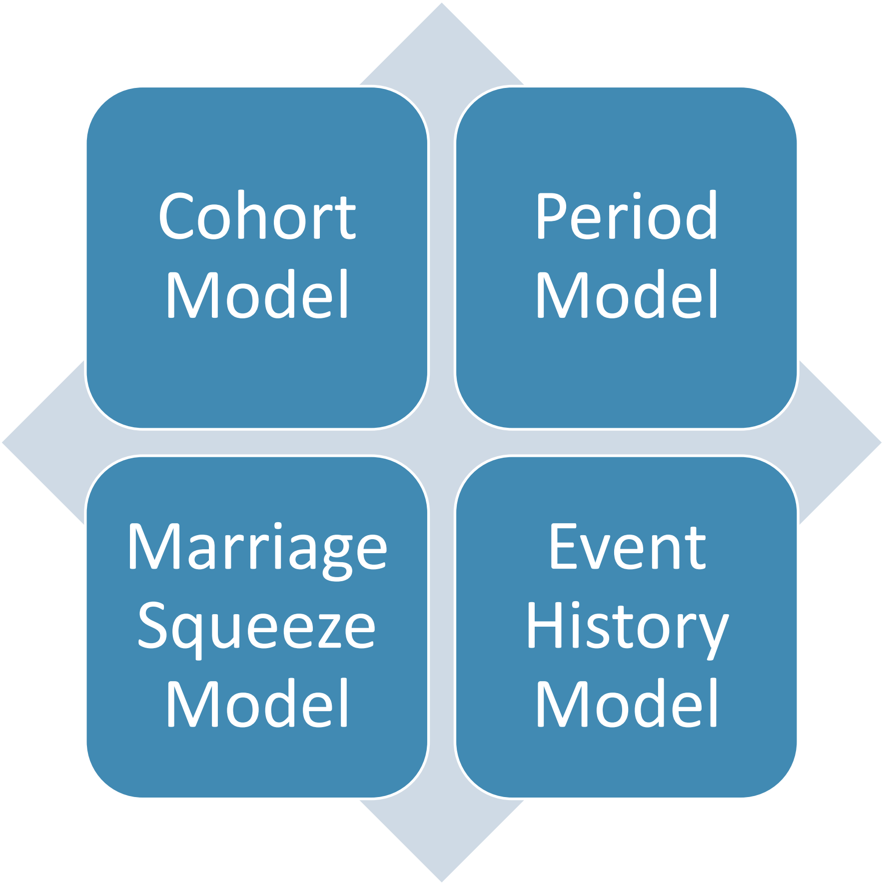
Polyamory

Group marriage, also known as polyamory, is a marriage system where multiple people form a union, and all individuals in the group are considered married to each other. Unlike polygamy, group marriage typically involves equal partnership among all members.

Key Characteristics:

- Several individuals form a mutual relationship, with romantic or sexual involvement with each other.
- All members are legally and socially recognized as partners, though such marriages are not recognized in most legal systems.

Models of Nuptiality



Estimating Mean Age at Marriage

The Mean Age at Marriage (MAAM) refers to the average age at which individuals marry for the first time.

It is a critical indicator used by demographers to assess societal changes such as delayed marriage, shifting gender roles, and the effects of socio-economic factors on marriage.

There are several methods used to estimate the Mean Age at Marriage, each with its advantages and limitations.

Direct Method (Cross-Sectional Data)

The Direct Method involves using cross-sectional data from surveys or censuses to calculate the mean age at marriage. This method is involves calculating the average age of individuals at the time of marriage for all those who have ever married.

$$\text{Mean Age at Marriage (MAAM)} = \frac{\sum \text{Age at Marriage}}{\text{Total Number of Marriages}}$$

Limitations:

- The data collected may not reflect the marriage patterns of younger generations who may delay marriage. As a result, this method may not fully account for shifts in marriage trends over time.
- It provides a snapshot, meaning it may not account for changes in behavior over a person's lifetime (i.e., young individuals may not yet have married).

Indirect Method- (Singulate Mean Age at Marriage - SMAM)

The Indirect Method for estimating the Mean Age at Marriage (MAAM), commonly referred to as **Singulate Mean Age at Marriage (SMAM),** is a widely used demographic technique to estimate the average age at marriage.

This method is particularly useful when detailed age-at-marriage data is unavailable, but marital status distributions by age group are available, such as in census or survey data.

The SMAM is based on the proportion of the population that is married at each age and calculates the average age at first marriage.

Unlike direct methods that require data on each individual's age at first marriage, the SMAM uses marital status data across age groups to derive the mean age at marriage.

Steps for calculating SMAM

1. **Step 1: Sum the unmarried proportions** for ages 15-49 (S_1 to S_7) and multiply by 5 (the number of years in each age group). This gives the total number of years individuals remain unmarried before age 50.

$$\sum_{i=15}^{49}(S_i \times 5)$$

2. **Step 2: Add the correction factor 1500** to the sum from Step 1. This compensates for the years lived unmarried before age 50 for a cohort of 100 individuals.

$$1500 + \sum_{i=15}^{49}(S_i \times 5)$$

3. **Step 3: Calculate the unmarried proportions** for individuals aged 45-49 (S_7) and 50-54 (S_8). These proportions will be used to adjust the final result.

4. **Step 4: Adjust for older age groups (45-49 and 50-54)** by multiplying their unmarried proportions by 50 to account for the underrepresentation of individuals who marry later in life. Subtract this value from the result of Step 2.

$$\left(\sum_{i=45}^{49} S_i + \sum_{j=50}^{54} S_j\right) \times 50$$

5. **Step 5: Calculate the total unmarried proportions** for the 45-49 and 50-54 age groups. This will be used for normalization in the denominator.

$$\sum_{i=45}^{49} S_i + \sum_{j=50}^{54} S_j$$

6. **Step 6: Divide** the result from Step 4 by the result from Step 5, and then multiply by 100 to get the **Singulate Mean Age at Marriage (SMAM)**.

The formula for calculating the **Singulate Mean Age at Marriage (SMAM)** is as follows:

$$\text{SMAM} = \frac{\left[\sum_{i=15}^{49}(S_i \times 5) + 1500\right] - \left[\sum_{i=45}^{49} S_i + \sum_{j=50}^{54} S_j\right] \times 50}{\left[\sum_{i=45}^{49} S_i + \sum_{j=50}^{54} S_j\right]} \times 100$$

Where:

- S_i represents the age-specific proportions of individuals who have not married at age i.
- The summation $\sum_{i=15}^{49}(S_i \times 5)$ sums the unmarried proportions between ages 15 and 49, each multiplied by 5 (assuming the data is categorized into 5-year age intervals).
- 1500 is a correction factor added to adjust the overall result.
- The subtraction term considers the unmarried proportions of individuals aged 45-49 and 50-54, multiplied by 50 to adjust for the older, potentially unrepresented age groups.
- The denominator accounts for the unmarried proportions in the age groups 45-49 and 50-54, providing normalization.

Limitations of SMAM

The method assumes that the **rate of marriage** at each age group is constant over time, which may not always be the case, particularly in populations with rapidly changing marriage trends.

Requires **age-specific marital status data**, which might not always be readily available in some regions or countries.

The method does not account for **cohabitation** or **informal unions**, which may be a significant form of partnership in certain societies.

NUPTIALITY TABLE

An extension of the standard life table that incorporates increments and decrements such as first marriage, first divorce, second marriage, second divorce, and so on.

Nuptiality tables may deal with marriage or marital dissolution or combination of both.

Gross Nuptiality Table

A Gross Nuptiality Table provides a summary of the marriage rates for a population, often grouped by age. It shows the number of marriages occurring within specific age groups, but it does not account for the age distribution of the population. Essentially, it reflects the overall marriage frequency without adjusting for the number of people at risk (single persons in each age group).

Assumptions:

- The table assumes that marriage rates are constant for each age group over time.
- It does not take into account mortality rates, migration, or other demographic factors that may influence the size of the unmarried population.
- It simply gives the total number of marriages occurring in each age group without considering the size of the eligible population.

Net Nuptiality Table

A Net Nuptiality Table is similar to the Gross Nuptiality Table but with an important difference—it adjusts the marriage rates by accounting for the number of unmarried individuals at risk. It often incorporates the effects of mortality and migration, giving a more accurate picture of how marriage rates impact the unmarried population over time.

Assumptions:

- Net Nuptiality accounts for both marriage and mortality, indicating the actual number of marriages occurring relative to the population at risk (i.e., the population of unmarried individuals).
- This table often assumes that migration does not significantly alter the unmarried population at each age.
- It provides a clearer picture of the pace at which a population of unmarried persons is decreasing over time due to marriage and death.

Key Nuptiality Measures or Indicators in Bangladesh

- Crude Marriage Rate
- General Marriage Rate
- Age Specific Marriage Rate
- Order Specific Marriage Rate
- First order specific Marriage Rate
- Total Marriage Rate
- Total First Marriage Rate
- Singulate Mean Age at Marriage

Migration

Migration is the movement of people from one geographical location to another, either temporarily or permanently, with the intention of settling, working, or residing in the new location.

- It can occur within a country (internal migration) or between countries (international migration), and it may be voluntary (in search of better opportunities) or forced (due to conflict, disasters, or persecution).

A migrant is an individual who changes their usual place of residence for a specified period (commonly at least 6 months or 1 year, depending on national or UN statistical definitions), crossing either administrative boundaries (internal) or national borders (international).

Push and Pull Factors of Migration

Push Factors

Push factors are the negative conditions or circumstances in the place of origin that compel or force people to leave their homes. These can be economic, social, political, or environmental in nature.

-They "push" people away from their current location.

Pull Factors

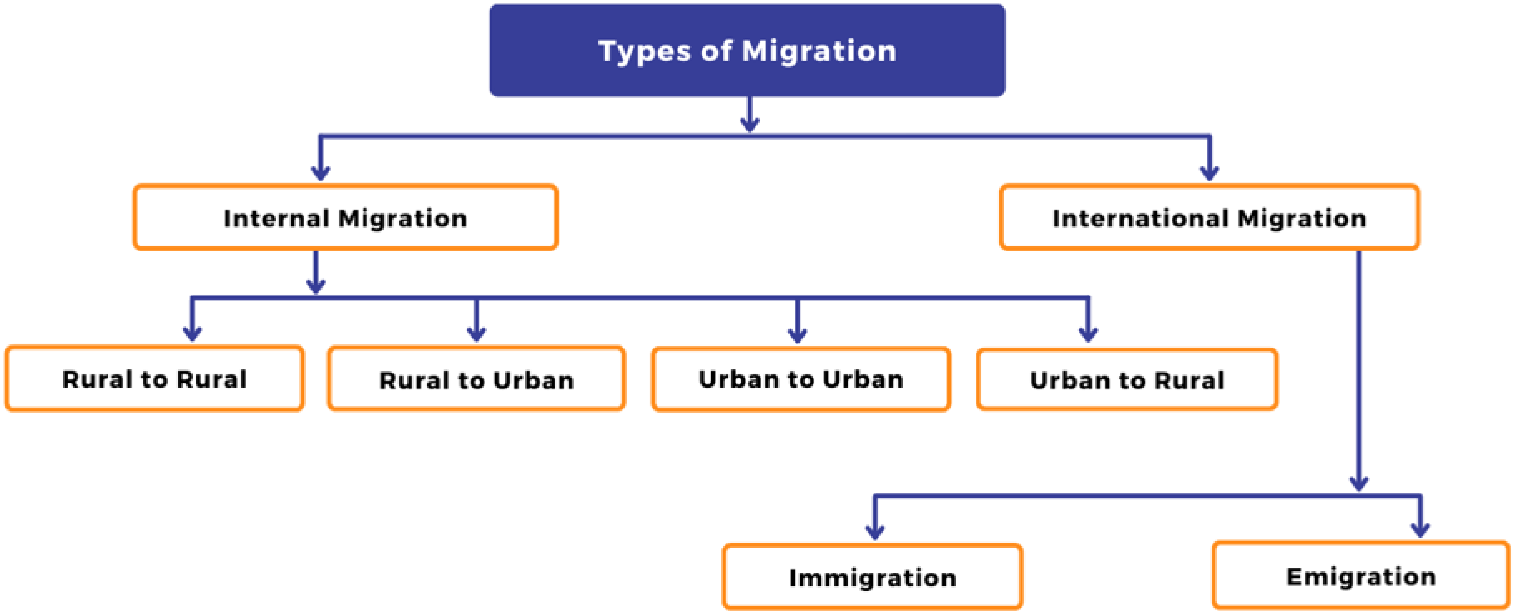
Pull factors are the positive conditions or opportunities in a destination area that attract people to move there.

-They "pull" people toward a new location.

Types and Examples of Push & Pull Factors

Category	Push Factors (from Origin)	Pull Factors (to Destination)
1. Economic	- Unemployment - Low wages - High cost of living - Lack of land or business opportunities	- Availability of jobs - Higher wages - Better standard of living - Access to credit or capital
2. Social	- Discrimination (ethnic, gender, religion) - Lack of education or healthcare - Poor social mobility	- Access to quality education - Better healthcare - Tolerance and inclusiveness - Social networks or family ties
3. Political	- War and conflict - Political repression - Corruption - Lack of freedom	- Political stability - Protection of rights - Rule of law - Democratic governance
4. Environmental	- Natural disasters (flood, drought, earthquake) - Climate change - Soil degradation or desertification	- Favorable climate - Fertile land - Safer environment - Disaster-resilient area
5. Demographic	- Overpopulation - High dependency ratio - Youth bulge with no jobs	- Aging population (demand for workers) - Low population density - Urbanization
6. Technological	- Lack of access to modern technology or internet - Limited transport or communication	- Technological advancement - Access to internet, smart cities, innovation hubs
7. Cultural	- Cultural oppression - Lack of freedom of expression or religion	- Cultural freedom - Opportunities for artistic or religious expression

Types of Migration



Internal Migration

Movement of people within the boundaries of a single country, without crossing international borders.

Subtype	Definition	Example
Rural to Urban	Movement from rural areas (villages) to urban areas (cities or towns) for better opportunities.	A farmer moving to Dhaka for work in a garment factory.
Urban to Rural	Movement from urban areas to rural areas, often for retirement or lifestyle changes.	Retirees moving from Dhaka to a rural area for a quieter life.
Urban to Urban	Movement between urban areas for career advancement, better living conditions, or education.	An engineer moving from Chittagong to Dhaka for a job.
Rural to Rural	Movement between rural areas, often related to seasonal or agricultural labor.	A farm worker moving to a different village during harvest season.

International Migration

The movement of people across national borders for work, residence, education, or refuge.

Subtype	Definition	Example
Emigration	Leaving one's home country to settle in another.	A Bangladeshi worker emigrating to Saudi Arabia for construction work.
Immigration	Moving into a new country to live, work, study, or seek refuge.	An Afghan refugee immigrating to Germany to escape war.

In-Migration

Movement of people into a specific area or region, leading to an increase in the population of that area.

Subtype	**Definition**	**Example**
Internal In-Migration	Movement within the same country from one region to another.	People moving from rural Bangladesh to Dhaka for better job opportunities.
International Immigration	Movement from one country to another for residence, work, or refuge.	A Bangladeshi family immigrating to Canada for a better quality of life and education.

Out-Migration

Movement of people leaving a specific geographic area, reducing the population of that area.

Subtype	Definition	Example
Internal Out-Migration	Movement from one region within a country to another.	People leaving Dhaka for quieter rural areas after retirement.
International Emigration	Movement from one's home country to another country for work, study, or refuge.	A worker from Bangladesh emigrating to the Middle East for employment.

Net Migration

- The difference between the number of people moving into an area (in-migration) and the number of people leaving the area (out-migration).
- Positive net migration means a population gain, while negative net migration indicates a population loss.
- Net Migration=In-Migration−Out-Migration

Subtype	Definition	Example
Positive Net Migration	More people moving into an area than leaving.	Dhaka experiencing population growth due to rural-to-urban migration.
Negative Net Migration	More people leaving an area than moving in.	Rural areas in Bangladesh having negative net migration as people leave for better opportunities in cities or abroad.

Measuring Migration

Migration Rate	Detailed Definition	Functional Form
Migration Rate (MR)	Overall movement of people in and out of a region relative to the total population.	$\frac{\text{Total Migrants (In + Out)}}{\text{Total Population}} \times 1000$
In-Migration Rate (IMR)	Measures the rate of people entering a region relative to its total population.	$\frac{\text{In-migrants}}{\text{Total Population}} \times 1000$
Out-Migration Rate (OMR)	Measures the rate of people leaving a region relative to its total population.	$\frac{\text{Out-migrants}}{\text{Total Population}} \times 1000$
Net Migration Rate (NMR)	Difference between the number of people moving into and out of a region, expressed per 1,000 people.	$\frac{\text{Immigrants}-\text{Emigrants}}{\text{Mid-year Population}} \times 1000$
Crude Migration Rate (CMR)	Total level of migration (both in and out) relative to the population.	$\frac{\text{Total Migrants in a Year}}{\text{Mid-year Population}} \times 1000$
Gross Migration Rate (GMR)	Total migration flow (in-migrants + out-migrants) relative to the population.	$\frac{\text{In-migrants + Out-migrants}}{\text{Total Population}} \times 1000$
Lifetime Migration Rate	Measures the percentage of the population that has ever migrated from their birthplace to another location.	$\frac{\text{People Who Have Migrated}}{\text{Total Population}} \times 100$
Recent Migration Rate	Measures the percentage of the population that has migrated within a defined recent period, such as the last 5 years.	$\frac{\text{People Who Migrated in the Last 5 Years}}{\text{Total Population}} \times 100$
Crude Migration Ratio (CMR)	Measures overall migration relative to total population per 1000 people.	$\frac{\text{Total Migrants in a Year}}{\text{Total Population}} \times 1000$

Migration Effectiveness Index (MEI)

The **Migration Effectiveness Index** (MEI) measures the balance between the number of people moving into a region (in-migrants) and the number of people moving out (out-migrants). It indicates how imbalanced or balanced the migration flow is, where a high MEI suggests a significant imbalance (either a large number of people coming in or leaving), and a low MEI indicates a more balanced migration flow.

$$\text{MEI} = \frac{|\text{In-migrants} - \text{Out-migrants}|}{\text{In-migrants} + \text{Out-migrants}} \times 100$$