CHAPTER 11

POPULATION PROJECTION

11.1 Introduction

A **population projection** is defined as a "best-guess" calculation of the number of people expected to be alive at a future date, based on what we know about the current population size and what we expect to happen to births, deaths, and migration. More formally, population projection is the technique of forecasting the demographic scenario of a region or a country that may be expected to unfold in future years. The forecasting is made possible by calculating the expected number of persons age by age for each sex at points in time subsequent to a census or other staring point. The process begins by specifying what the future trends in birth, death and migration are likely to be. This is followed by a series of calculations, which spell out the amount of population change that would result if these conditions were to materialize. The set of birth, death and migration rates used may be those of some past period, or they may be an extrapolation from the past; the extrapolation may follow a mathematically specified method or it may be intuitive or an educated guess. Because the forecast is completely determined by the stated assumptions, demographers refer to the results of these calculations as **projections**.

Population projections are always set on a "conditional" future because we can never be certain about the assumptions we use in the projection. Projections illustrate possible courses of population change based on assumptions about future births, deaths, net international migration, and domestic migration. In some cases, several series of projections are produced based on alternative assumptions for future fertility, life expectancy, net international migration, and (for state-level projections) state-to-state or domestic migration.

Projections made under realistic assumptions are called '**Forecasts**'. It is important to remember that forecasts are usually no more than educated guesses or conjectures dressed up in sophisticated ways. If the assumed guesses are in error, then the forecast will be misleading. Moreover, the 'more' one forecasts into the future, the 'more' are the assumptions likely to be wrong. Thus, forecast usually leads to be reasonable for a short period, but then become progressively worse – the margin of error increases.

We sometimes talk of population estimates. While projections and estimates may appear similar, there are some distinct differences between the two measures. **Estimates** are for the recent past, near future and more often for intercensal periods, while projections are based on assumptions about future demographic trends. Estimates generally use existing data collected from various sources, while projections make assumptions about what demographic trends will be in the future. An estimate is unlikely to be based on valid assumptions on the future courses on the components (e.g. fertility, mortality and migration) of population changes.

In literature, there are primarily two approaches of making population projection: mathematical method and component method. In **mathematical method**, evolution of the population is assumed to be described by some fairly simple mathematical formulae closed to the concept of simple statistical models. In **component method**, the components of population change, viz. the fertility, mortality and migration, are taken into consideration explicitly. We describe these two approaches in the next two sections.

11.2. Mathematical Method of Projection

In mathematical method of population projection, no attempt is made to model the components of population change explicitly. Normally, the method is applied to the total population. In mathematical method, there are only three commonly used methods: arithmetic, geometric, and exponential growth methods. In addition, logistic law of growth is also employed for the projection of population.

The simplest way of projecting a population under mathematical approach is to assume that the population size at time 't' is a function of the population size at time '0', called initial population, Algebraically,

$$P_t = f(P_0) \qquad \text{.... (11.1)}$$

The equation (11.1) is dependent upon the rate of growth with which population P_0 increases to P_t during the time interval 0 to t of length t years.

11.2.1 Arithmetic Projection

The simplest approach to estimating the population at some future date is achieved through what is known as the **linear** or **arithmetic projection.** This approach essentially assumes that the absolute change in the population per unit time is constant. The ultimate equation for such projection is of the form

$$P_t = P_0(1+rt) \quad \text{... (11.2)}$$

where

P_0 = Size of the initial population at time 0.
P_1 = Size of the population at time t.
t = Length of interval for which growth is computed
r = Rate of linear growth during the interval (0, t) of length t.

The estimating equation for any intermediate point in time between 0 and t derived from (11.2) is given by

$$P_x = P_0 + \left(\frac{x}{t}\right)(P_t - P_0) \quad \text{.... (11.3)}$$

where x is the number of years from base year to the year of estimate. The equation (11.3) is essentially a result of linear interpolation of population size at a point x between P_0 and P_t.

We illustrate with an example how the equation (11.2) and (11.3) are used in estimating the population size at some future date.

Example 11.1: The population of Bangladesh as enumerated in 2011 census was 142315 thousand (Preliminary Results, Bangladesh Population Census, 2011). The average annual **linear growth** rate during the intercensal period 2001–2011 was estimated to be .0142 per person per annum based on an intercensal interval of 10 years and 51 days. If this rate of growth is assumed to remain constant over the next 10 years, what would be the size of the population in 2021?

Using (11.2) for t=10, we have

$$P_{2021} = P_{2011}(1+rt) = 142319(1+.0142\times 10) = 162528 \text{ thousand.}$$

What will be the expected size of the population 5 years later following 2011?

Referring to the above example and using (11.3), the estimated size of the population after 5 years (i.e. in 2016) from the base year (i.e. 2011) is

$$P_{2016} = P_{2011} + \left(\frac{5}{10}\right)(P_{2021} - P_{2011})$$

$$= 142319 + \frac{1}{2}(162528 - 142319)$$

$$= 152424$$

The same size of the population may be arrived at simply by using the equation (11.2). For example

$$P_{2016} = P_{2011}(1 + rt)$$

$$= 142319(1 + .0142 \times 5)$$

$$= 152424$$

If the same rate of growth can be assumed to hold for the period 2001-2011, then an estimated size of the population five years back (a backward projection) may be obtained using a formula of the type

$$P_0 = \frac{P_t}{(1 + rt)}$$

This formula is essentially a rearrangement of the formula (11.2). Applying this equation, we estimate the population for 2006 using 2011 as the base year

$$P_{2006} = \frac{P_{2011}}{(1 + rt)} = \frac{142319}{(1 + .0142 \times 5)}$$

$$= \frac{142319}{1.017} = 132884$$

11.2.2 Geometric Projection

If the rate of growth is assumed to be constant and is thought of as occurring at discrete points of time, the size of the population (P_t) at some future time 't' is estimated by using the formulas (11.4) as shown below:

$$P_t = P_0(1 + r)^t \qquad \text{... (11.4)}$$

The equation (11.4) represents a method, which is called **geometric projection** and r is the geometric rate of growth. For Bangladesh population, the geometric rate of growth was estimated to be .0134 for the intercensal period 2001–2011. Applying this rate to the 2011 population, an estimate for 2021 was arrived at:

$$P_{2021} = P_{2011}(1+r)^t = 142319(1+.0134)^{10} = 162582$$

The geometric law of growth may also be used to project the population backward or even between two consecutive census dates at some specified point of time assuming an appropriate rate of growth. If we wish, for example, to estimate the population for 2006 using 2011 population as the base year population, then the geometric law of growth with a geometric rate of growth of .0134 yields

$$P_{2006} = \frac{P_{2011}}{(1+.0134)^5} = \frac{142319}{(1+.0134)^5} = 133155$$

If a linear rate of growth (.0142) is employed in geometric projection, the estimated population size comes out to

$$P_{2006} = \frac{P_{2011}}{(1+.0142)^5} = \frac{142319}{(1.0142)^5} = 132631$$

11.2.3 Exponential Projection

A more realistic assumption frequently made in projecting a population at some future date is that the population growth during the projection period is continuous in time. The projection under this assumption is known as **exponential projection** and is achieved through the following formula:

$$P_t = P_0 e^{rt} \qquad \text{... (11.5)}$$

Logically, r is known as the exponential rate of growth. After a little rearrangement of the equation (11.5), we arrive at the following formula for computing exponential rate of growth:

$$r = \frac{\log_e p_t - \log_e p_0}{t} \qquad \text{... (11.6)}$$

Using the exponential model of growth as in equation (11.5), the estimated size of the population in 2021 with base year population of 2011 is

$$P_{2021} = P_{2011} e^{.0134\times10} = 142319(1.14339) = 162727.$$

11.2.4 Polynomial Projection

If population data are available for a sufficiently long period of time in the form of time series data, it is sometimes rewarding to fit a quadratic or higher degree curve to the data of the form:

$$P_t = a + bt + ct^2 \qquad \text{... (11.7)}$$

The results obtained from fitting a curve of this type are usually more accurate than if a linear interpolation or extrapolation is used. The equation yields reasonably good estimates of the population for the intercensal periods or immediate post-censal periods. To illustrate the method, let us employ the census populations of Bangladesh as shown in the accompanying table:

Table 11.1: Census Population of Bangladesh: 1901–2011

Census year	t	Enumerated population (in million)
1901	0	28.9
1911	1	31.6
1921	2	33.3
1931	3	35.6
1941	4	42.0
1951	5	44.2
1961	6	55.2
1971	7	71.5*
1981	8	87.1
1991	9	106.3
2001	10	130.5
2011	11	147.9

*This figure actually refers to 1974, but for illustrative purpose, we use this as an estimate of 1971

Use of least-squares method gave the following equations:

$$535.7 = 10a + 45b + 285c$$

$$3079.2 = 45a + 285b + 2025c$$

$$21937.6 = 285a + 2025b + 15333c$$

Solving the equations, a=31.95, b=–3.03 and c= 1.24

Thus the estimated equation is

$$P_t = 31.95 - 3.03t + 1.24t^2.$$

We employ the fitted equation to estimate the population of 2011 and 2021 which correspond to two census years of Bangladesh.

A value of t=11corresponds to the projected year 2011, so that the estimated size of the population in 2011 using the estimated equation is 148.66 million:

$$P_{2011} = 31.95 - 3.03(11) + 1.24(11)^2 = 148.66$$

while the census of 2011 reported an adjusted population of 149.8 million.

For projecting the total population at the next census date in 2021, t=12 and consequently

$$P_{2021} = 31.95 - 3.03(12) + 1.24(12)^2 = 174.15.$$

It is thus estimated that in 2021, the Bangladesh population will stand at 174.15 million.

11.2.6 Logistic Law of Growth

When the rate of growth is greater than '0' and the time interval for which the projection is made, is long, application of either the geometric or the exponential method results in ridiculously large figures, in which case, a more realistic approach would be to use a mathematical function, which produces an upper bound, such as **logistic curve**, first proposed by Velhust and later by Pearl and Reed. The formula for the logistic curve derived from the exponential function (11.5) is of the form:

$$P_t = \frac{r}{Cre^{-rt} + k} \qquad \text{... (11.8)}$$

where C is a constant associated with an initial population size (C is not the same as P_0). Equation (11.8) assumes an *S*-shaped curve for P_t with an upper asymptote of r/k. Figure 11.1displays such a curve.

In its simplest form, the logistic curve described by (11.8) is as follows:

$$y = \frac{k}{1 + e^{\alpha + \beta t}} \qquad \text{... (11.9)}$$

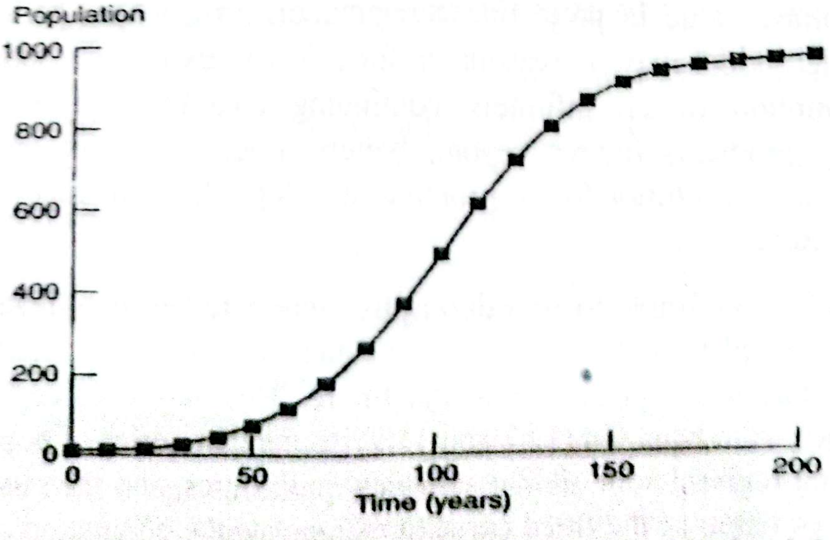

Figure 11.1: A logistic curve drawn for (11.7) for r=.05, k=.00005 and C=.148

This curve has been observed to fit to the growth of some species of animals and some bacterial cultures, which usually grow rapidly at the beginning when placed in limited environment with adequate supply of food and space for their initially few members. This population then grows slowly as the population approaches a point, where there is pressure on available resources. The curve ultimately approaches an upper limit to the number that can be maintained, whereupon the population ceases to grow, and thus the *S*-shape.

Despite its usefulness, the mathematical method has several limitations. A few of them are listed below:

a) The method implicitly assumes that the same model will continue to apply throughout the projection period. Population, however, never grows in such regular fashion.

b) The method does not take into consideration the current levels and trends in the components of population change.

c) The method seldom uses historical evidences pertaining to the components of population growth.

d) Most mathematical methods tend to overestimate future trends in population.

e) When applied to projecting the size of several sub-groups within the same population, the mathematical method ignores the logical interdependences among these sub-groups.

In particular, the pitfalls in using either the arithmetic formula or geometric formula are numerous. Decidedly, population of humans does not increase linearly nor it does so entirely geometrically. Thus these two approaches are of limited value in projecting a population for a longer period. The exponential model may be reasonable for a few years into the future, but the assumption of an infinitely continuing constant rate of growth eventually produces figures beyond beliefs. Logistic law of growth is nearly a realistic solution to this problem, which produces an upper limit of the population.

It is therefore advisable to fit a descriptive model rather than those based on mathematical formula to the existing data and then use the fitted model to extrapolate the population into the future. This may involve fitting a curve (such as in equation (12.7 and 11.8) to the time series of population totals using regression or similar statistical procedures, and then using the resulting equation of the fitted curve to extrapolate the population at some desired future date. This procedure also suffers from the limitation of attaching equal weight to each data point used. Weighted moving average is sometimes used to get rid of this problem in that each projected figure is taken into account when projecting the next.

11.3 Component Method of Projection

A somewhat more realistic approach of estimating future population size is to make reasonable assumption about the patterns of changes in the components of population growth; namely the changes in the rates of fertility, mortality and migration. Estimates made in this way are known as the **component projection**. This procedure is based upon the ideas underlying the balancing equation that we have discussed earlier:

$$P_t = P_0 + (B - D) + \overline{M} \qquad \text{...(11.10)}$$

where,

P_0 = Population sizes at the beginning of the projection interval;

P_t = Population at terminal year (t);

B =Births during the projection interval (0, t);

D =Deaths during the projection internal (0, t);

$\overline{M}$ =Net migration during the projection interval (0, t).

This equation just breaks population changes down into its individual components. Now, this equation can be applied not only to the aggregate population but also to its constituent components, by age, sex, marital status, etc. A very useful variant of component method of projection is the so called cohort component method that we discuss below

11.4 Cohort Component Method of Projection

In the cohort-component method, the components of population change (fertility, mortality, and net migration) are projected separately for each birth cohort (persons born in a given year). The base population is advanced each year by using projected survival rates and net international migration. Each year, a new birth cohort is added to the population by applying the projected fertility rates to the female population.

To carry out any component projection requires the followings:

(a) A base population with which the projection starts by age and sex.
(b) A set of age-specific fertility rates in the base year.
(c) A set of age-sex specific mortality rates in the base year in the form of survival ratios.
(d) A set of age-sex specific migration rates.

The cohort component method of projection was developed by P.K. Welpton of the Scripps Foundation for Research on Population Problem of the Miami University.

As actually practiced, population projections are usually performed on age-sex specific basis. Fertility is expressed as fertility by age of mothers. The force of mortality is expressed as 'survival ratios' or the proportion of persons surviving from one age to a later age. The component of migration is expressed as the amount of net migration classified by age and sex.

The calculations are made in terms of '**projection cycles**' each of which is usually five years in length. Starting with a census or other base date, the demographer applies the fertility and migration schedules for the duration of one cycle and then summarizes his results to obtain an estimate of the population as of the date that makes the end of the cycle. The population at the end of the cycle, obtained from these operations, then becomes the base population for a second projection cycle. The projection cycle is repeated to obtain an estimate for a date one interval further into the future. Additional estimating cycles are carried out until the future date for which an estimate is desired is reached.

The procedure is very much flexible. The demographer may use a different schedule of fertility, mortality and migration at each cycle. In making his choice on the future courses of these growth components, the demographer must exercise his knowledge on the subject. He must ask himself: will the fertility/mortality/migration remain constant during the projection period? If not, how much change is anticipated?

The demographers thus must have an answer to the above questions and will schedule his future courses of fertility, mortality and migration accordingly.

Cohort component method is widely used because it provides a flexible and powerful approach to population projection. Dividing the population into age cohorts is an important methodological innovation. It permits the analysts to account for differences in mortality, fertility and migration rates among different age groups and to consider how these rates change over time. The components of population changes (viz. fertility, mortality and migration) need to be distinguished from each other while making use of these components in population projection. First, such distinctions enable us to account separately for the demographic causes of population changes. Second, each component of change typically responds differently to changes in economics, social, political, cultural, medical environmental and other factors that affect population changes. Finally, the behavior of each component varies among places and follows different trends over time.

11.4.1 Steps Involved in Cohort Component Method

The computational procedure for the component method of projection is a straightforward application of the balancing equation on an age specific basis. Steps involved in the population projection may be enumerated as follows:

1. Establish the base population from which the estimates will begin

The estimating procedure must start from a base population that is usually sub-divided into 5-year age-groups by sex. Since demographic rates are usually computed from mid-year populations, as of July 1 in a year in which a population census has been taken, mid-year population usually provides the most appropriate base.

2. Make allowance for mortality that will occur among the population during the projection cycle.

This step is primarily necessary to project the population to the next cycle through the life table survival rates. These ratios measure the probability that a person in a particular five-year age-group will be alive five-years later and be a member of the next older five-year age group.

The equation for these survival ratios is:

$$_5S_x = \frac{_5L_{x+5}}{_5L_x} \qquad \text{... (11.11)}$$

These ratios are readily computed from an abridged (five-year) life table, which is to be selected to represent average mortality conditions during the projection cycle. The number of persons of each age group who survive to the end of the cycle is estimated by multiplying for each age group, the base population by the survival ratio. Thus for projecting the population of age group 0–4, we proceed as follows:

$$P_{0-4} \times {_5S_0} = P_{5-9} \qquad \text{... (11.12)}$$

where P_{5-9} is the population of 5-9 age group for the projected year five-year later, etc.

Similarly, to project the population of age group 5-9, we require estimating the survival ratio $_5S_5$:

$$ {}_5S_5 = \frac{{}_5L_{x+10}}{{}_5L_{x+5}} \quad \ldots(11.13) $$

and hence

$$ P_{5-9} \times {}_5S_5 = P_{10-14} \quad \ldots(11.14) $$

In absence of a reliable life table, model life tables (e.g. UN Model Life Table, Coale-Demeny Regional Model Life Table, Brass Two-parameter Life Table, and South Asian Model Life Table) may be chosen. The simplest approach to determine the future course of mortality is to assume that the trend of mortality exhibited in the recent past will continue in the future. It is often advisable to specify a set of target mortality rates as a lower limit. The expectation of life at birth is a good indicator of mortality pattern for any population. A careful examination of the life expectancy at birth will lead us to determine the future course of mortality and hence a plausible choice of a life table.

The choice of the future trends in expectation of life may be based on the UN suggestions (DESA, 1955). Examination of mortality trends in several countries led the United Nations to conclude that for countries where the expectation of life at birth was under 55, the annual increase in life expectancy has been about 0.5 years. When this is between 55 and 65, the annual increase is somewhat greater. The annual increase tended to decline when expectation of life at birth exceeded 65 years. This guideline may not however work well if the population experiences more rapid decline in mortality, thus resulting in higher expectation of life at birth. Nevertheless, the UN recommendation is a useful guide for the projection of mortality for many developing countries. Once the assumption regarding future mortality is determined, it is a straightforward task to use the mortality trends in cohort component projection.

3. Make allowance for net migration during the projection cycle

The anticipated amount of net migration for each age group is determined to represent the amount of population gain or loss expected for each age-group as a result of net migration during the cycle.

In many countries, migration statistics at the national level are not available. Even if available, these statistics are unreliable and inaccurate. Hence net migration is sometimes estimated indirectly. This makes the projection questionable. It is because of this reason, most projections assume closed population, which implicitly assumes that the population for which the projection is made, is closed against migration (i.e. no in and no out migration).

4. Make allowance for fertility

During each projection cycle a certain number of infants will be born. The number of births that occur will depend upon the age-specific fertility rates that are expected to prevail during that cycle. The objective of this step is to forecast the age-specific fertility rates that will characterize each future projection cycle and to apply these rates to the women of child-bearing age to obtain the projected number of births.

In most instances, population projections are undertaken by postulating not one but several patterns of change in fertility. For example, one can assume that in a given projection interval from the present, fertility will either remain constant as it is at the present time, reduced by 25 percent, or reduced by 50 percent. The first assumption requires that a constant schedule of age-specific fertility rates reflecting current fertility levels be applied to the population in each age group for each projection interval. The second and third assumptions (reduction of 25 and 50 percent) require that the fertility schedule be so adjusted that it reflects the course of the fertility decline.

The simplest method of projecting fertility is based on assumed changes in the general fertility rate (GFR). Under this approach, the average GFR for each projection interval is applied to the number of women exposed to the risk of child-bearing to determine the number of births occurring during the interval. What then constitutes the population of women exposed to the risk of child-bearing to which the GFR is to be applied? A neat solution to the problem is to apply the average population of females at the beginning and end of the projection interval. The result, when multiplied by the GFR and length of the projection interval, yields an estimate of the newborns occurring during the 5- year interval. The following formula is used to arrive at the estimate of births:

$$ {}_nB_t = \frac{n}{2}\left({}_{35}W_{15,t} + {}_{35}W_{15,t+n}\right) GFR \qquad \text{... (11.15)} $$

where

n = Length of the projection interval

${}_{35}W_{15,t}$ = Number of women aged 15–49 at the beginning of the interval

${}_{35}W_{15,t}$ = Number of women aged 15–49 at the end of the interval.

${}_nB_t$ = Number of births to be estimated for the interval (t, $t+n$)

To obtain the new population in the youngest age group (0–4 usually) requires only the application of the appropriate survival ratio to these new births. In general:

$$ {}_nP_{0,t} = {}_nB_t\left(\frac{{}_nL_0}{nl_0}\right) \qquad \ldots (11.16) $$

When the projection is made by sex, the formula will be revised accordingly to take into account the sex.

For estimation of births during the projection interval, age-specific fertility rates may also be used though application of such rates is more complex. Age-specific fertility rates at each projection interval are applied to the population exposed to the risk of childbearing to determine the total number of births taking place in the interval. As before, the average population at the beginning and end of the projection interval is used to estimate the births. This population is determined as follows:

$$ \overline{W}_x = \frac{{}_nW_{x,t} + {}_nW_{x,t+n}}{2} \qquad .. (11.17) $$

This mid-interval population of women as obtained in (11.17) then can be used to make an estimate of the number of births during the interval.

$$ {}_nB_t = \frac{n\sum\left({}_nW_{x,t} + {}_nW_{x,t+n}\right)}{2}\,{}_nf_x \qquad \ldots (11.18) $$

where

${}_nB_t$ = Number of births to be estimated for the interval (t, $t+n$)

${}_nW_{x,t}$ = Number of women aged x to $x+n$ at the beginning of the projection interval

${}_nW_{x,t+n}$ = Number of women aged x to $x+n$ at the end of the projection interval.

${}_nf_x$ = Annual age-specific fertility rates.

For scheduling age-specific fertility and mortality rates at some future dates, some prefers to use Lee-Carter model (Lee and Carter, 1992). For fertility projection, for example, the model is of the type:

$$ \log_e(f_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \qquad \ldots (11.19) $$

where

a_x = Average age-specific pattern of fertility;

k_t = Time trend index of general fertility;

b_x = Pattern of deviations over age as k_t varies;

$\varepsilon_{x,t}$ = Error term;

$f_{x,t}$ = Fertility rate at age x at time t.

The Lee-Carter model is a numerical algorithm used in mortality forecasting and life expectancy forecasting. The input to the model is a matrix of age specific mortality rates ordered monotonically by time, usually with ages in columns and years in rows. The output is another forecasted matrix of mortality rates.

The Lee-Carter model was introduced by Ronald D. Lee and Lawrence Carter in 1992 with the article "Modeling and Forecasting the Time Series of U.S. Mortality," (Journal of the American Statistical Association 87 (September): 659-671). The model grew out of their work in the late 1980s and early 1990s attempting to use inverse projection to infer rates in historical demography. The model has been used by the United States Social Security Administration, the US Census Bureau, and the United Nations. It has become the most widely used mortality forecasting technique in the world today.

5. Sub-divide births by sex and survive births to the next projection cycle

Because the population projection is done separately for males and females, the total births projected must be sub-divided into male and female births. The person making this population projection must specify the proportion of all births which he expects to be male (the remainder will automatically be female). By multiplying this total number of births that can occur each year by this proportion, the numbers of male and female births that occur each year can be known.

The births anticipated in step 4 will not all survive to the next projection cycle. An estimate of the number of births that will survive to the next cycle is thus necessary. This is achieved by multiplying these births by an appropriate survival ratio, taken from the life table. The resulting number is the total population aged 0-4 at the beginning of the next cycle. The sub-division and survival of the births are accomplished through the application of the following formula for estimating female population at the youngest age group:

$$ {}_nW_0 = \frac{n\sum\left({}_nW_{x,t} + {}_nW_{x,t+n}\right)}{2}\left({}_nf_x\right)\left(\frac{{}_nL_0^f}{n\,l_0}\right)\left(p^f\right) \qquad \text{... (11.20)} $$

where p^f is the proportion of births that were females. Note that the survival ratio used above is taken from a female life table. For projection of the male population, both the survival ratios and the proportion of births will be different.

$$\frac{n\sum\left({}_nW_{x,t}+{}_nW_{x,t+n}\right)}{2}\left({}_nf_x\right)\left(\frac{{}_nL_0^m}{n l_0}\right)\left(p^m\right) \qquad \ldots (11$$

where p^m is the proportion of births that were males such that $p^m +$ $p^f = 1$.

6. Consolidate the results of the preceding steps.

Present your projection results separately by age and sex in a single tabl

7. Begin a new projection cycle.

The projection cycle is repeated to obtain an estimate for a date o interval further into the future. Additional estimating cycles are carried o until the future date for which an estimate is desired is reached.

The steps described above may well be visualized from the following flo chart:

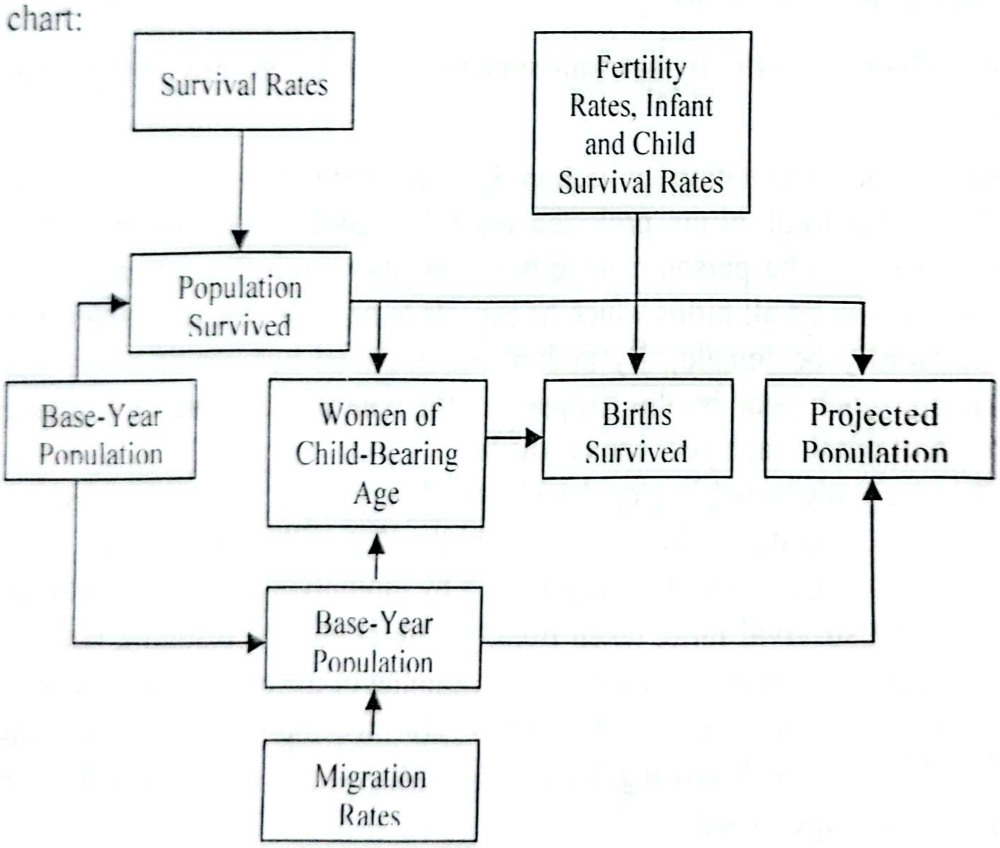

Figure 11.2: Flow chart for component method of projecting population

11.5 An Illustrative Example

We now illustrate the projection by component method first with th female population followed by male population of Bangladesh as obtaine in 2001 census. The projection is done in two cycles each of five-yea interval. Since no firm data on migration are available, we will assume th the population is closed against international migration, both in and out. The base population adjusted for under-count and age-heaping errors is used for the projection purpose (see 2001 Census Report, Volume 1, Table 1.7, page:20).

The projection of the population in the present case will be carried out for five-year age groups. Use of five-year age groups has the advantage of the fact that the members of a particular five-year age group may be treated as a cohort which, five years later, will constitute the next older five-year age groups. By using five-year survival ratios, the members of each cohort may be 'aged' progressively from cycle to cycle.

The reference date of the census is January 23, 2001. It is a usual practice to use the base population as on the first day of July in which case the base population is to be shifted backward or forward to match the mid-year (i.e. July 1) population. This may be done first estimating the total population as on 1 July by some suitable means (e.g. employing geometric or exponential formula) and simply prorating this population by age and sex to the desired date. In prorating, the age structure of the enumerated (or adjusted) population is employed. The method of prorating an age distribution has been illustrated in Chapter 2. We make no such attempt here because of the fact that our purpose is merely illustrative. The first cycle will be of five years and thus the projected population will bear the reference date as on January 23, 2006.

We begin the exercise with the female population first in five-year age groups. In turn we will discuss the way in which mortality and fertility components have been used in the projection work.

11.5.1 Mortality Assumptions

The simplest possible assumption about the future courses of mortality is that it will remain unchanged during the projection cycle. This is nearly valid only when the projection interval is small. For Bangladesh population, such an assumption for a five year period is not very unrealistic given the current trends and patterns of mortality. In such case, the survival ratios may be derived from any reliable life table at the base year. For Bangladesh, no such life table is available for 2001. However, if such tables are not available, one can use model life table on the basis of some suitably chosen mortality indicator. Expectation of life is one such good indicator of mortality. Bangladesh Bureau of Statistics reports an expectation of life of 64.5 years in 2001 for females (see Sample Vital Registration Report, 2001, Table 7.1, Page 109). We employ this value to choose an appropriate model life table from a set of Regional Model Li Tables due to Coale and Demeny. The female life expectancy of 64 years, rounded to 65 years corresponds to the mortality level 19 in th aforesaid model life table constructed for West region, known as W Model life table. This table contains a set of five-year survival rat (Lingner, 1974).

Table 11.1 below shows the base population for females in column (2) an the female survival ratios in column (3). The expected numbers survivors are arrived at simply multiplying the base population by th survival ratios. For example, the projected population in 2006 in the ag group 5–9 is the product of population of 2001 in 0–4 and the surviv ratio .9851:

$$P_{0-4} \times \frac{{}_5L_5}{{}_5L_0} = 7967 \times .9851 = 7848$$

The subsequent calculations are done in a similar way for each age grou to arrive at the 2006 population.

The computation of the last population figure at age 65+ in 2006 need explanation. This is to be computed using T_x values of the life table rathe than L_x values and is just T_{70}/T_{65}. At level 19 in West Model, this ratio fo females is .6563. The 2006 population aged 65+ then comprises th survivors of both the 60–64 years olds and the 65+ olds in 2001. Thus th numbers of survivors in 2006 at 65+ is

$$(1372 \times .8815) + (2309 \times .6563) = 2725$$

In algebraic notations,

$$_{\infty}W_{65,t+5} = \left({}_5W_{60,t} \times \frac{{}_5L_{65}}{{}_5L_{60}} \right) + \left({}_{\infty}W_{65} \times \frac{T_{70}}{T_{65}} \right) \quad \ldots 11.2$$

11.5.2 Fertility Assumption

What remains now is the estimation of the population in the 0–4 age grou in 2006. This population is the survivors of the births that took plac during the projection interval 2001–2006. To accomplish this, we need set of current fertility rates. The assumptions about fertility that need to b made may be quite simple or very detailed depending essentially on th level of details required in the final projection results. For the first cycle o projection, we employ the age-specific fertility schedule for 2001–200 period obtained in 2004 BDHS (see BDHS Report, 2011, Page 64, Tabl 5.3.2). These rates (see Table 11.2) when multiplied by the number o women exposed to the risk of child-bearing will yield the number of births occurring during the projection cycle 2001–2006.

11.5.3 Projection of Female Population

Now the question is: what constitutes the population to which each age-specific fertility rate is to be applied? The simplest approach to defining this population is to estimate the female population aged x to $x+n$ at the middle of the time period (in our present case, 2001–2006), by averaging the populations in these ages at the beginning and end of the projection interval:

$$\overline{W}_x = \left(\frac{{}_5W_{x,t} + {}_5W_{x,t+5}}{2} \right) \quad \text{... (11.22)}$$

The average number of women obtained for our projection employing (11.22) is shown in column 5 of Table 11.2.

Table 11.2: Projection of Female Population of Bangladesh for Five-Year Period: 2001–2006

Age	Base population (2001)	Five year–survival ratio	Projected population (2006)	Average females population 15–49
$(x, x+5)$	${}_5W_{x,t}$	${}_5L_{x+5}/{}_5L_x$	${}_5W_{x,t+5}$	$\overline{W}_x$
(1)	(2)	(3)	(4)=(2)×(3)	(5)= [(2)+(4)]/2
0–4	7967	.9851	**8405**[b]	–
5–9	8268	.9939	7848[a]	–
10–14	7805	.9932	8218	–
15–19	6026	.9904	7752	6889
20–24	6546	.9879	5968	6257
25–29	6233	.9859	6467	6350
30–34	4464	.9833	6145	5305
35–39	3769	.9795	4389	4079
40–44	2910	.9735	3692	3301
45–49	2131	.9635	2833	2482
50–54	1884	.9480	2053	–
55–59	1106	.9228	1786	–
60–64	1372	.8815	1021	–
65+	2309	–.	2725	–
Total	**62791**		**69302**	–

Source: Base population, 2001 Bangladesh Census Report Volume 1: Table 1.7, Page 20

a: 7848=7967×.9851, **b**. 8405 = 8915 × .9428

This population at middle of the interval can then be used to develop an estimate of the number of births during the five-year interval:

$$_5B_t = 5\sum\left(\overline{W}_x \times {}_5f_x\right) \qquad \text{... (11.24)}$$

The estimated number of births for both sexes combined obtained thus is shown in Table 11.3.

Since we are projecting the female population, the estimated births must be split into male and female births. If we have the female age-specific fertility rates based on the female births, such conversion will not be needed. Under the given circumstances, we segregate the total births into male and female births assuming that to every one female birth, there are 1.05 male births. This results in a proportion equal to 1/2.05, which tells us that out of every 2.05 births, there will be one female birth. Thus under this assumption, the number of female births for the five-year period $_5B_t^f$ is obtained as follows:

$$_5B_t^f = \frac{\text{Toatl number of births in 5 years}}{1+1.05}$$

$$= 5\sum\left(\overline{W}_x \times {}_5f_x\right) p^f$$

$$= 5(3655)\left(\frac{1}{2.05}\right) = 8915$$

Note that the factor $p^f = \frac{1}{2.05}$ is the proportion of births that were females. The numerator 3655 is multiplied by 5 because five years of births are required in the projection.

It is important to note that not all of these births will survive the entire projection interval. An appropriate survival ratio now is needed for these newborns. The same model life table corresponding to Level 19 with an expectation of life at birth of 65 years (see Lingner, 1975: Handbook for Population Analysts, Part B) is used to estimate the survival of these newborns until 2006. The survivorship ratio is $_5L_0 / 5l_0$ =.9428. Thus the projected number of surviving female children in the youngest age group at the end of the projection period is estimated at

$$_5W_0 = {}_5B_t^f\left(\frac{_5L_0}{5l_0}\right) = 8915 \times .9428 = 8405$$

This estimate is entered in column (4) of Table 11.2 as the first entry against the age 0–4. This completes the projection of female population for the year 2006. The complete results of this projection is shown in Table 11.2

Table 11.3: Age-Specific Fertility Rates: BDHS, 2001–03 and the Estimated Number of Births

Age x	Average number of women $\overline{W}_x$	Age specific fertility rates (2001–2003) ${}_5f_x$	Annual number of total births ${}_5B_t = \overline{W}_x \times {}_5f_x$
15–19	6889	.135	930
20–24	6257	.192	1201
25–29	6350	.135	857
30–34	5305	.083	440
35–39	4079	.041	167
40–44	3301	.016	53
45–49	2482	.003	7
Total	**34663**	**3.025***	**3655**

Source: ASFRs, BDHS Report, 2011, page 64, Table 5.3.2, * TFR

11.5.4 Projection of Male Population

The projection of the male population for the first cycle is analogous to the projection of female population described in section 11.5.3. A choice of an appropriate model life table constructed for males and estimation of male children will lead us to such a projection. The same model life table source is used here corresponding to an expectation of life of 61.23 years. This is however an arbitrary choice in the face of uncertainty of the expectation of life at birth but yet we believe this is our best guess. The required data and computational steps are shown in Table 11.4. The estimated number of male births is arrived at as follows:

$${}_5B_t''' = 5\sum\left(\overline{W}_x \times {}_5f_x\right) p''' = 5(3655)\left(\frac{1.05}{2.05}\right) = 9360$$

The projected number of surviving males in the youngest age group 0–4 at the end of the projection period is estimated as

$${}_5M_0 = {}_5B_t'''\left(\frac{{}_5L_0'''}{5l_0}\right) = 9360 \times .9300 = 8705$$

This estimate is inserted in column (4) of Table 11.4 as the first entry against the age 0–4.

Table 11.4: Projection of Male Population of Bangladesh for Five-Year Period: 2001–2006

Age group (x, x+5)	Base population (2001) $_5M_{x,t}$	Five-year survival ratio $_5L_{x+5}/_5L_x$	Projected population (2006) $_5M_{x,t+5}$
(1)	(2)	(3)	(4)=(2)×(3)
0–4	8913	.9826	8705
5–9	9270	.9929	8758[a]
10–14	8783	.9918	9204
15–19	6720	.9878	8711
20–24	5246	.9853	6638
25–29	5307	.9840	5169
30–34	4576	.9809	5222
35–39	4456	.9753	4489
40–44	3629	.9658	4346
45–49	2742	.9508	3505
50–54	2275	.9277	2607
55–59	1383	.8933	2111
60–64	1602	.8433	1235
65+	2829	–	3123[b]
Total	**67731**	–	**73851**

Source: Base population, 2001 Bangladesh Census Report Volume 1: Table 1.7, Page 20

a: 8758=8913×.9826, b: $3123 = (1602 \times .8433) + (2829 \times .6263)$.

11.6 Projection of Population for the Second and Subsequent Cycles

If the population is desired to be projected beyond five years, the same procedures may well be followed under varieties of assumptions regarding fertility, mortality and migration. The following are some of the frequently adopted assumptions of the future courses of the demographic components:

1. High variant (or constant variant): Under this variant, it is assumed that the base year rates of mortality and fertility will remain constant during the second projection cycle.

2. Medium variant: Here we assume that fertility will decline but mortality will remain constant or that fertility will remain constant and mortality will decline.

3. Low variant: Here we assume that both fertility and mortality will decline.

To project the population for the next 5 years 2006–2011, we can choose any of the variants as outlined above and proceed to project the population following the procedures just described for the projection of the population for the first cycle.

The simplest assumption of future courses of fertility is that it will remain constant throughout the projection interval. If there is reason to believe that fertility is declining, one can give an effect to this decline after thorough examination of the past trends in fertility.

A review of the population policy of Bangladesh Govt. shows that the sixth population policy of Bangladesh envisages to achieve replacement level of fertility in terms of total fertility rate (TFR) of 2.1 by three targeted years: 2011, 2016 and 2021 (Census Report, Volume–I, Page 173). Keeping this target in view, BBS projects the population under the alternative targets of reaching replacement level of fertility. Following the target of achieving TFR=2.1 by the year 2011, we reschedule the age-specific fertility rates of the new base year to match this value just by prorating the TFR. The prorating factor k is as follows:

$$k = \frac{\text{Assumed TFR}}{\text{Baseline TFR}} = \frac{2.1}{3.025} = .6942$$

Assumptions similar to fertility may be adopted for mortality as well. For shear reason of simplicity and illustrative purposes, we retain the same set of model life table functions and choose e_0 (males)=61.23 years and e_0 (females)=65 years that corresponds to west mortality level 19 in projecting the population for the second cycle.

The survival ratios from the chosen life table and the expected number of survivors in 2011 are shown in Table 11.5. The estimated number of female population in the reproductive age groups are also furnished in column 5 of the table under reference.

The initial set of age-specific fertility rates, when multiplied by k, yields the projected fertility rates to be used in estimating the number of births in the second projection interval. These rates are shown in column 3 of Table 11.6.

Table 11.5: Projection of Female Population of Bangladesh for Five-Year Period: 2006–2011

Age group $(x, x+5)$	Base population (2006) ${}_5W_{x,t}$	Five year–survival ratio ${}_5L_{x+5}/{}_5L_x$	Projected population (2011) ${}_5W_{x,t+5}$	Average females population 15–49 $\overline{W}_x$
(1)	(2)	(3)	(4)=(2)×(3)	(5)= [(2)+(4)]/2
0–4	8405	.9851	**6428**	–
5–9	7848	.9939	8280	–
10–14	8218	.9932	7800	–
15–19	7752	.9904	8162	7957
20–24	5968	.9879	7678	6823
25–29	6467	.9859	5896	6182
30–34	6145	.9833	6376	6261
35–39	4389	.9795	6042	5216
40–44	3692	.9735	4299	3996
45–49	2833	.9635	3594	3214
50–54	2053	.9480	2730	–
55–59	1786	.9228	1946	–
60–64	1021	.8815	1648	–
65+	2725	–.	2688[a]	–
Total	**69302**	–	**73561**	–

Source: Base population, 2001 Bangladesh Census Report Volume 1: Table 1.7, Page 20

a: $2688 = (1021 \times .8815) + (2725 \times .6563)$.

Table 11.6: Projected Age-Specific Fertility Rates and Estimated Number of Births: Second Projection Cycle

Age group x	Average number of women $\overline{W}_x$	Age specific fertility rates ${}_5f_x$	Annual number of total births ${}_5B_t = \overline{W}_x \times {}_5f_x$
(1)	(2)	(3)	(4)
15–19	7957	.094	748
20–24	6823	.133	907
25–29	6182	.094	581
30–34	6261	.058	363
35–39	5216	.028	146
40–44	3996	.011	44
45–49	3214	.002	6
Total	**39649**	[illegible]	[illegible]

$$5(2795)\left(\frac{1}{2.05}\right)=6817$$

Thus the projected numbers of surviving female children in the youngest age group at the end of the projection period is estimated as

$${}_5W_0 = {}_5B_t^f\left(\frac{{}_5L_0^f}{5l_0}\right) = 68178 \times .9428 = 6428$$

The projection of male population now follows a routine work. The number of male births and the survival of these births to the next projection period are shown below:

Total number of male births:

$${}_5B_t^m = 5\sum\left(\overline{W}_x \times {}_5f_x\right)\, p^m = 5(2795)\left(\frac{1.05}{2.05}\right) = 7158$$

Number of male survivors:

$${}_5M_0 = {}_5B_t^m\left(\frac{{}_5L_0}{5l_0}\right) = 7158 \times .9300 = 6657$$

The results of the projection of male population are shown in Table 11.7.

Table 11.7: Projection of Male Population of Bangladesh for Five-Year Period: 2006–2011

Age	Base population (2006)	Five year–survival ratio	Projected population (2011)
$(x, x+5)$	${}_5M_{x,t}$	${}_5L_{x+5}/{}_5L_x$	${}_5M_{x,t+5}$
(1)	(2)	(3)	(4)=(2)×(3)
0–4	8705	.9826	**6657**
5–9	8758	.9929	8554
10–14	9204	.9918	8696
15–19	8711	.9878	9129
20–24	6638	.9853	8605
25–29	5169	.9840	6540
30–34	5222	.9809	5086
35–39	4489	.9753	5122
40–44	4346	.9658	4378
45–49	3505	.9508	4197
50–54	2607	.9277	3333
55–59	2111	.8933	2418
60–64	1235	.8433	1886
65+	3151	–	3015[a]
Total	**73851**	–	**77616**

Source: Base population, 2001 Bangladesh Census Report Volume 1: Table 1.7.

a: $3015 = (1235 \times .8433) + (3151 \times .6263)$.

The finally projected populations for the two cycles are shown in Table 11.8.

Table 11.8: Projected Population for Bangladesh: 2001–2011

	First cycle: 2006			Second cycle: 2011		
Age group	Male	Female	Total	Male	Female	Total
0–4	8705	8405	17110	6657	6428	13085
5–9	8758	7848	16606	8554	8280	16834
10–14	9204	8218	17422	8696	7800	16496
15–19	8711	7752	16463	9129	8162	17291
20–24	6638	5968	12606	8605	7678	16283
25–29	5169	6467	11636	6540	5896	12436
30–34	5222	6145	11367	5086	6376	11462
35–39	4489	4389	8878	5122	6042	11164
40–44	4346	3692	8038	4378	4299	8677
45–49	3505	2833	6338	4197	3594	7791
50–54	2607	2053	4660	3333	2730	6063
55–59	2111	1786	3897	2418	1946	4364
60–64	1235	1021	2256	1886	1648	3534
65+	3151	2725	5876	3041	2688	5729
Total	73851	69302	143153	77616	73561	151187

11.7 Matrix Method of Population Projection

The techniques of cohort component method of population projection can be compactly written in matrix notations as has been established by Benardelli (1941), Lewis (1942) and especially Lesie (1945). Later Kefitz (1968) presented the process of projections in terms of a matrix of fertility and mortality transition probabilities and a vector representing the age distribution of the population.

In matrix method, the projection is carried out by multiplying a matrix in which the first row contains the fertility factors, survival factors for women of child-bearing age, and the survival factors for the first age group, while the remaining rows of the matrix contain the survival factors for all ages by a vector of ${}_nW_x$ values.

The projection matrix is of the following form:

$$
\begin{pmatrix}
0 & 0 & \frac{p_5L_0}{2l_0}\left(\frac{p_5L_{15}}{{}_5L_{10}}\,{}_5f_{15}\right) & \frac{p_5L_0}{2l_0}\left({}_5f_{15}+\frac{p_5L_{15}}{{}_5L_{10}}\,{}_5f_{20}\right) & \ldots 0 & 0 \\
\frac{{}_5L_5}{{}_5L_0} & 0 & 0 & 0 & \ldots 0 & 0 \\
0 & \frac{{}_5L_{10}}{{}_5L_5} & 0 & 0 & \ldots 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & \ldots & \ldots & \ldots & \ldots \frac{{}_5L_{10}}{{}_5L_5} & 0
\end{pmatrix}
\begin{pmatrix}
{}_5W_{0,t} \\ {}_5W_{5,t} \\ {}_5W_{10t} \\ \ldots \\ \cdot \\ {}_5W_{85t}
\end{pmatrix}
$$

11.8 A Note on the Assumption on Projection

The projection assumptions regarding the future courses of fertility, mortality and migration are usually made by the experts who are entrusted with the responsibility of preparing projections. In almost all national projections, projections are made under three different variants labelled high, medium and low. No doubt, the labeling of the variants is a relative one. The 'high variant' employs the highest fertility level resulting in the highest future population size, while the 'low variant' results in the lowest population size at some future date. The 'medium variant' falls between the high and the low variants. The assumptions on the future courses of mortality are relatively difficult. In most instances the mortality assumption is made with reference to the expectation of life. Due to paucity of migration data, it is a usual practice to assume that the population is closed against migration. In projecting the population of Bangladesh for a period of 50 years, from 2011 to 2061, for example, Geoffrey Hayes (2015) considered three different scenarios with regard to the future courses of fertility and mortality. Under the high variant, Hayes assumed that the base year total fertility rate (2.3) will remain constant during the entire projection cycle. This is the assumption under high variant. This assumption will result in a projected population size of 265.2 million in 2061. Under the medium variant assumption, the TFR will fall from its base year rate of 2.3 to 1.9 in 2016 resulting in a projected ... 2061. Under the low variant the base expectation of life has been assumed to increase from 70.3 years to 78.9 in 2061allowing for the sex differences in the expectation.

11.9 Importance and Uses of Population Projection

Why do we make population projections? Population projections are useful for a number of reasons and help stakeholders plan for the near and distant future. If we know how many people are in a country or region, this puts us in a better position to assess the need for new jobs, teachers, schools, doctors, nurses, urban housing, food, and requirements for resources. For example, in order to plan an immunization program at some time in the future, governments, donors, and healthcare staff need to know how many children will be alive in the future. Population projections can help us know future population size. Population projections are also important for raising awareness of issues among policymakers. For example, a population projection can help illustrate the impact of an increased population on the use of fuel wood and the potential threat to the forests or the need for affordable housing projection.

In sum, it is increasingly important to have high quality statistics on the population and projections of the population, for policy development and for planning and providing public services in different geographic areas. Specifically, the uses of population projection may be attributed to the following areas

(a) Forecasts of population composition;
(b) Distribution of the future population;
(c) Vital events that are likely to occur in future;
(d) Future requirements for community services;
(e) Size of the future labor force and the corresponding demand for employment;
(f) Central and local finance allocation;
(g) Informing local and national policy;
(h) Housing and land use planning;
(i) Health care planning;
(j) Modeling and projecting health care indicators;
(k) Benchmarking other projections;
(l) Looking at the implications of an ageing population; and
(m) Making national and international comparisons, etc.

and many others.