Confounding:

The information on some unimportant treatment combinations may be mixed up with the incomplete block differences in the replicates. This device is known as confounding.

Thus, confounding is a process by which one or more factorial effects are deliberately entangled with block effects in a replicate consisting of several blocks of equal size.

Types of confounding:

There are two important types of confounding, namely, which are defined with example as follows:

i) Total confounding or complete confounding.

ii) Partial confounding.

Total confounding:

If the set of effects and interactions are confounded in all the replicates, it is said to be completely confounded. Total confounding involves complete loss of information on the confounded effects.

Example:

Let us consider an experiment with factors A, B and C each at two levels.

The treatment combinations are:-

	(1)	a	b	ab	c	ac	bc	abc
$For\ ABC:$	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$+$

It may be desirable to have incomplete blocks of 4 units in each replicate and the effect of ABC interactions are confounded with the block effects.

$Replication-1$

B_1	B_2
abc	(1)
a	ab
b	ac
c	bc

ABC -Confounded

$Replication-2$

B_1	B_2
c	ac
a	bc
b	ab
abc	(1)

ABC -Confounded

$Replication-3$

B_1	B_2
a	ab
abc	(1)
c	ac
b	bc

ABC -Confounded

So, this is a completely confounded factorial experiment.

Partial confounding:

If different sets of interactions are confounded in separate replications, then those are said to be partially confounded. In a partial confounding, confounded effects are estimated from the replicates in which they are individually unconfounded.

Example:

In a 2^3-factorial experiment with factors A, B and C. The ABC interaction may be confounded in the first replicate, AB in the second replicate, AC in the third replicate. For this the block compositions are as follows-

Treatment Combination :	(1)	a	b	ab	c	ac	bc	abc
For ABC :	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$+$
For AB :	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$
For AC :	$+$	$-$	$+$	$-$	$-$	$+$	$-$	$+$

Replication -1

B_1	B_2
a	(1)
b	ab
c	ac
abc	bc

ABC - Confounded

Replication -2

B_1	B_2
(1)	a
ab	b
c	ac
abc	bc

AB -Confounded

Replication -3

B_1	B_2
(1)	a
b	ab
ac	c
abc	abc

AC - Confound

In the above arrangement ABC is confounded with blocks in 1^{st} replicate but is unconfounded in the other two replicates. So ABC can be estimated from 2^{nd} and 3^{rd} replicate. AB is confounded in 2^{nd} replicate only and AB can be estimated from 1^{st} and 3^{rd} replicates. Similarly AC is confounded with 3^{rd} replicates only and AC can be estimated from 1^{st} and 2^{nd} replicates in which AC is unconfounded. Thus ABC, AB, AC are partially confounded and partial information on ABC, AB and AC are obtained from the replicates in which they are individually unconfounded.

Besides these there are other kinds of confounding which are-

i) Balanced confounding.

ii) Unbalanced confounding.

Balanced confounding:

If all effects of a certain order, say the $2-$factor interactions are confounded with incomplete block differences equal number of times, the confounding is called balanced partial confounding, and the resulting design is called partially balanced design.

Unbalanced confounding:

If all the contrasts of a certain order are confounded unequal numbers of times, the confounding is known as unbalanced partial confounding and the resulting design is called unbalanced partial design.

Characteristics of confounding technique:

The technique of confounding is used in factorial experiments with the following characteristics.

- **i)** Block size is less than the number of whole sets of treatment combinations.
- **ii)** Block differences are eliminated from error variation so that adequate control of experimental errors is achieved.
- **iii)** Relatively unimportant factorial effects are entangled with block effects so that information on such least important effects is lost and sacrificed.
- **iv)** Other important factorial effects are measured with improved procession within the blocks.

Describe the confounding techniques:

The confounding technique consists of:

- **i)** Choosing certain factorial effects, usually high order interactions for confounding.
- **ii)** Then dividing the whole set of treatment combinations into two or more groups of equal size by some prescribed rule of one group having all positive signs and the other group having all negative signs for a selected **factorial** effect and these groups must be balanced with respect to main effects and lower order interaction.
- **iii)** Thereafter randomly allocating the treatment combinations of these groups to blocks of replication such that those selected unimportant effects can not be separated from incomplete block effects. Hence the chosen unimportant effects are said to be confounded with incomplete blocks.

Properties of confounding:

Confounding has the following properties: -

- i) Only predetermined interactions are confounded.
- ii) The estimates of interactions which are not confounded are orthogonal.

Advantage of confounded:

- i) In confounding factorial experiments, replicates are divided into nearly homogeneous incomplete blocks. Thus, confounding reduces block size and permits precise estimation of main effects and interactions from homogeneous blocks.

ii) Variation between incomplete blocks is obtained and it is eliminated and thus reduces error variation. Thus, confounding leads to adequate error control.

iii) Confounding arrangement involving reduced homogeneous blocks makes the application of RBD and LSD feasible and quite satisfactory.

Disadvantages:

i) If entails definite loss of information on confounded effects.

ii) Analysis of data in a confounded factorial experiment is relatively complicated.

iii) A great deal of inconvenience arises when treatments interact with incomplete blocks.

iv) Confounded effects are replicated less frequently than unconfounded effects.

Analyzing data for complete confounding in 2^3 – factorial experiment:

For 2^3 – factorial experiment having three factors A, B and C each at two levels $0, 1$. The treatment combinations are

	(1)	a	b	ab	c	ac	bc	abc
$For\ ABC:$	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$+$

The layout plan of block contents of the replication of 2^3 factorial experiment where ABC are confounded as:-

$Replication-1$ $\quad$ $Replication-2$ $\quad$ $Replication-3$

B_1	B_2
abc	(1)
c	ab
b	ac
a	bc

ABC -Confounded

B_1	B_2
c	ac
a	bc
b	ab
abc	(1)

ABC -Confounded

B_1	B_2
a	bc
abc	(1)
c	ac
b	bc

ABC -Confounded

The unconfounded effects and there SS can be computed conveniently by Yate's method.

Yates method:

Treat Combination	Total field	Column-1	Column-2	Column-3	Main & Interaction effect	SS
(1)	x_1	$x_1+x_2=y_1$	$y_1+y_2=z_1$	$z_1+z_2=p_1$	-	
a	x_2	$x_3+x_4=y_2$	$y_3+y_4=z_2$	$z_3+z_4=p_2$	$A=\frac{p_2}{4r}$	$SS(A)=\frac{p_2^2}{2^3 r}$
b	x_3	$x_5+x_6=y_3$	$\vdots$	$\vdots$	$B=\frac{p_3}{4r}$	$SS(B)=\frac{p_3^2}{2^3 r}$
ab	x_4	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
c	x_5	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
ac	x_6	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
bc	x_7	$\vdots$	$\vdots$	$\vdots$	$BC=\frac{p_7}{4r}$	$SS(BC)=\frac{p_7^2}{2^3 r}$
abc	x_8	$x_8-x_7=y_8$	$y_8-y_7=z_8$	$z_8-z_7=p_8$		

Since interaction ABC has been confounded with incomplete blocks, ABC is not determined here and it SS is executed from treatment SS.

Analysis of data in RBD:

$$Total\ SS = \sum_{i=1}^{r}\sum_{j=1}^{2^3} y_{ij}^2 - \frac{G^2}{2^3 r}$$

$$Block\ SS = \sum_{i=1}^{2r} \frac{B_i^2}{4} - \frac{G^2}{2^3 r}$$

$$Treatment\ SS = \sum_{j=1}^{2^3} \frac{T_j^2}{r} - \frac{G^2}{2^3 r} - SS(ABC)$$

$$= SS(A) + SS(B) + \dots\dots\dots + SS(BC)$$

$$Error = Total\ SS - Block\ SS - Treatment\ SS$$

ANOVA TABLE

Sources of variation	$D.F$	SS	MS	F
$Blocks$	$2r-1$	S_b^2	$s_b^2 = \frac{S_b^2}{2r-1}$	
$Treatments$	6	S_t^2	$s_t^2 = \frac{S_t^2}{6}$	
A	1	S_A^2	$s_A^2 = \frac{S_A^2}{1} = S_A^2$	s_A^2 / s_e^2
B	1	$\vdots$	$\vdots$	$\vdots$
AB	1	$\vdots$	$\vdots$	$\vdots$
C	1	$\vdots$	$\vdots$	$\vdots$
AC	1	$\vdots$	$\vdots$	$\vdots$
BC	1	$\vdots$	$\vdots$	s_{BC}^2 / s_e^2
$Error$	$6(r-1)$	S_e^2	$s_e^2 = \frac{S_e^2}{6(r-1)}$	
$Total$	$2^3(r-1)$			

Hypothesis testing:

H_0 : All the unconfounded main effects and interactions are insignificant.

H_a : They are significant.

CR is $F_{cal} \ge F_{1,6(r-1)}(\alpha)$

Similarly any other factorial effect can be confounded if denied and accordingly data can be analyzed.

Analyzing data for complete confounding in 2^4 – factorial experiment:

For 2^4 – factorial experiment having four factors $A,\ B,\ C$ and D each at two levels $0\ \&\ 1$. The treatment combinations are-

Treatment Combination:	(1)	a	b	ab	c	ac	bc	abc	d
For $ABCD$:	$+$	$-$	$-$	$+$	$-$	$+$	$+$	$-$	$-$
	ad	bd	abd	cd	acd	bcd	$abcd$		
	$+$	$+$	$-$	$+$	$-$	$-$	$+$		

The layout plan of block contents of the replication of 2^4 factorial experiment where $ABCD$ are confounded are (assuming the experiment are replicated three times)-

Replication − 1

B_1	B_2
$abcd$	a
(1)	b
ab	c
ac	d
bc	abc
ad	abd
bd	acd
cd	bcd

$ABCD$ - Confounded

Replication − 2

B_3	B_4
ab	b
(1)	c
$abcd$	d
bc	a
ac	acd
ad	abc
bd	abd
cd	bcd

$ABCD$ - Confounded

Replication − 3

B_5	B_6
ab	abc
ac	abd
bc	acd
ad	bcd
bd	d
cd	c
(1)	b
$abcd$	a

$ABCD$ - Confounded

The unconfounded effects and their SS can be computed conveniently by Yates' method-

Yates method:

Treat Combination	**Total field**	**Column-1**	**Column-2**	**Column-3**	**Column-4**	**Main & Interaction effect**	SS
(1)	x_1	$x_1+x_2=y_1$	$y_1+y_2=z_1$	$z_1+z_2=p_1$	$p_1+p_2=q_1$	-	
a	x_2	$x_3+x_4=y_2$	$y_3+y_4=z_2$	$z_3+z_4=p_2$	$p_3+p_4=q_2$	$A=\frac{q_2}{(2^{4-1})r}$	$SS(A)=\frac{q_2}{2^4 r}$
b	x_3	⋮	⋮	⋮	⋮	$B=\frac{q_3}{(2^{4-1})r}$	$SS(B)=\frac{q_3}{2^4 r}$
ab	x_4	⋮	⋮	⋮	⋮	⋮	⋮
c	x_5	⋮	⋮	⋮	⋮	⋮	⋮
ac	x_6	⋮	⋮	⋮	⋮	⋮	⋮
bc	x_7	⋮	⋮	⋮	⋮	⋮	⋮
abc	x_8	⋮	⋮	⋮	⋮	⋮	⋮

d	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
ad	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
bd	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
abd	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
cd	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
acd	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
bcd	x_{15}	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$BCD=\frac{q_{15}}{\left(2^{4-1}\right)r}$	$SS\left(BCD\right)=\frac{q_{15}}{2^4 r}$
$abcd$	x_{16}	$x_{16}-x_{15}=y_{16}$	$y_{16}-y_{15}=z_{16}$	$z_{16}-z_{15}=p_{16}$	$p_{16}-p_{15}=q_{16}$		

Since interaction ABC has been confounded with incomplete blocks, ABC is not determined here and its SS is excluded from treatment SS .

Analysis of data in RBD:

$$Total\ \ SS\ \ =\sum_{i=1}^{r}\sum_{j=1}^{2^4} y_{ij}^2-\frac{G^2}{2^4 r}$$

$$Block\ \ SS=\sum_{i=1}^{2r}\frac{B_i^2}{8}-\frac{G^2}{2^4 r}$$

$$Treatment\ \ SS=\sum_{j=1}^{2^4}\frac{T_j^2}{r}-\frac{G^2}{2^4 r}-SS\left(ABCD\right)$$

$$=SS\left(A\right)+SS\left(B\right)+\dots\dots\dots+SS\left(BCD\right)$$

$$Error=Total\ \ SS-Block\ \ SS-Treatment\ \ SS$$

ANOVA TABLE

Sources of variation	$D.F$	SS	MS	F
$Blocks$	$2r-1$	S_b^2	$s_b^2=\frac{S_b^2}{2r-1}$	
$Treatments$	14	S_t^2	$s_t^2=\frac{S_t^2}{14}$	
A	1	S_A^2	$s_A^2=\frac{S_A^2}{1}=S_A^2$	$F_1=s_A^2/s_e^2$
B	1	$\vdots$	$\vdots$	$\vdots$
AB	1	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
BCD	1	S_{BCD}^2	s_{BCD}^2	

				$F_{14} = {s_{BCD}^2}/{s_e^2}$
$Error$	$14(r-1)$	S_e^2	$s_e^2 = \frac{S_e^2}{14(r-1)}$	
$Total$	$2^4(r-1)$			

Hypothesis Testing:

H_0 : All the unconfounded factorial effects are insignificant.

H_1 : They are significant.

CR is $F_{cal} \geq F_{1,14(r-1)}(\alpha)$

Q: Find the layout plan of block content of a replicant of 2^4 – factorial experiment where $ABCD$ & AC, BD is confounded. How do you find the ANOVA table for the analysis of the data?

Let us consider a 2^4 – factorial experiment with 4 factors A, B, C and D each at 2 levels. Then this experiment has $2^4 = 16$ possible treatment combinations which are used in standard order as shown bellow:

$Treatment\ Combination:$	(1)	a	b	ab	c	ac	bc	abc	d
$For\ ABCD:$	$+$	$-$	$-$	$+$	$-$	$+$	$+$	$-$	$-$
	ad	bd	abd	cd	acd	bcd	$abcd$		
	$+$	$+$	$-$	$+$	$-$	$-$	$+$		

Inorder to confound interaction $ABCD$ at first, the two balance groups are constructed such that one group contain treatment combinations with plus sign and the other group consists of treatment with minus sign which is shown bellow:

$Replication-1$

$ABCD$ -confounded

Inorder to confounded AC interaction along with $ABCD$ in this experiment. We divide each of these blocks into two blocks of 4 units and 4 blocks are ultimate obtain for one replication as follows-

B_1	B_2	B_3	B_4
(1) ac bd	ab bc ad	b abc d	a c abd

abcd	*cd*	*acd*	*bcd*

Here,

$$\left.\begin{array}{l}(B_1 \;\;\&\;\; B_2) \rightarrow +ve \;\; sign \\ (B_3 \;\;\&\;\; B_4) \rightarrow -ve \;\; sign\end{array}\right\} for \;\; ABCD \;\; confounded$$

Also

$$\left.\begin{array}{l}(B_1 \;\;\&\;\; B_3) \rightarrow +ve \;\; sign \\ (B_2 \;\;\&\;\; B_4) \rightarrow -ve \;\; sign\end{array}\right\} for \;\; AC \;\; confounded$$

Another method for determining block compositions or block contents is as follows:

Analysis of data in RBD:

$$Total \;\; SS = \sum_{i=1}^{r}\sum_{j=1}^{2^4} y_{ij}^2 - \frac{G^2}{2^4 r}$$

$$Block \;\; SS = \sum_{i=1}^{2r}\frac{B_i^2}{4} - \frac{G^2}{2^4 r}$$

$$Treatment \;\; SS = \sum_{j=1}^{2^4}\frac{T_j^2}{r} - \frac{G^2}{2^4 r} - SS(ABCD) - SS(AC) - SS(BD)$$

$$= SS(A) + SS(B) + \ldots \ldots \ldots + SS(BCD)$$

$$Error = Total \;\; SS - Block \;\; SS - Treatment \;\; SS$$

ANOVA TABLE

Sources of variation	*D.F*	*SS*	*MS*	*F*
Blocks	$4r-1$	S_b^2	$s_b^2 = \frac{S_b^2}{4r-1}$	
Treatments	12	S_t^2	$s_t^2 = \frac{S_t^2}{12}$	
A	1	S_A^2	$s_A^2 = \frac{S_A^2}{1} = S_A^2$	$F_1 = s_A^2 / s_e^2$
B	1	⋮	⋮	⋮
AB	1	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
BCD	1	S_{BCD}^2	s_{BCD}^2	$F_{14} = s_{BCD}^2 / s_e^2$
Error	$12(r-1)$	S_e^2	$s_e^2 = \frac{S_e^2}{12(r-1)}$	
Total	$2^4 r-1$			

Hypothesis Testing:

H_0: All the unconfounded factorial effects are insignificant.

$H_1:$ They are significant.

CR is $F_{cal} \geq F_{1,12(r-1)}(\alpha)$