

$$(a+b)^2 = a^2 + 2ab + b^2 \rightarrow a^2b^0 + 2a^1b^1 + a^0b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \rightarrow a^3b^0 + 3a^2b^1 + 3a^1b^2 + a^0b^3$$

$$\therefore (a+b)^4 \rightarrow a^4b^0 + a^3b^1 + a^2b^2 + a^1b^3 + a^0b^4$$

$$(a+b)^5 = (a+b)(a+b)(a+b)(a+b)(a+b)$$

$$= a^5b^0 + {}^5C_1 a^4b^1 + {}^5C_2 a^3b^2 + {}^5C_3 a^2b^3 + {}^5C_4 a^1b^4 + a^0b^5$$

$$(a+b)^4 = a^4b^0 + {}^4C_1 a^3b^1 + {}^4C_2 a^2b^2 + {}^4C_3 a^1b^3 + a^0b^4$$

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b^1 + {}^nC_2 a^{n-2}b^2 + {}^nC_3 a^{n-3}b^3 + \dots + b^n$$

* দ্বিপদী উপপাদ্যের বৈশিষ্ট্য:

(i) ঘাত n হলে পদসংখ্যা $n+1$

(ii) a এর ঘাত নিম্নমুখী হলে b এর ঘাত উচ্চমুখী।
 a এর ঘাত উচ্চমুখী হলে b এর ঘাত নিম্নমুখী।

(iii) সহগের মান বাড়তে বাড়তে একটি নির্দিষ্ট Number এ পৌঁছানোর পর একই প্যাটার্ন মানটি বারংবার করে কমতে থাকে।

* বিস্তৃত কর: $(a+2x)^5$

$$\Rightarrow (a+2x)^5 = a^5 + {}^5C_1 a^4(2x) + {}^5C_2 a^3(2x)^2 + {}^5C_3 a^2(2x)^3 + {}^5C_4 a(2x)^4 + (2x)^5$$

Subject : Date :

$$\Rightarrow (a+2x)^5 = a^5 + 5 \cdot {}^5C_1 a^4 x + 10 \cdot {}^5C_2 a^3 x^2 + 10 \cdot {}^5C_3 a^2 x^3 + 5 \cdot {}^5C_4 a x^4 + x^5 32x^5$$

সাধারন পদ

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \dots b^n$$

$$\text{এখানক ২য় পদ} = {}^nC_1 a^{n-1} b^1$$

$$= {}^nC_{(2-1)} a^{n-(2-1)} b^{(2-1)}$$

$$\text{৩য় পদ} = {}^nC_2 a^{n-2} b^2$$

$$= {}^nC_{(3-1)} a^{n-(3-1)} b^{(3-1)}$$

$$\text{মুতাবাঃ } r\text{তম পদ } f_r = {}^nC_{r-1} a^{n-(r-1)} b^{r-1}$$

$$r+1\text{তম পদ, } f_{r+1} = {}^nC_{r+1-1} a^{n-(r+1-1)} b^{r+1-1}$$

$$f_{r+1} = {}^nC_r a^{n-r} b^r$$

* $(a+b)^n$ এর বিস্তৃতিতে $r+1$ তম পদ, $f_{r+1} = {}^nC_r a^{n-r} b^r$

Math: $(2x^2 - \frac{3}{x})^{11}$ এর বিস্তৃতিতে x^0 এর সহগ নির্ণয় কর।

Subject :

Date :

⇒ এখানে

$$r+1 \text{ তম পদ, } x_{r+1} = {}^{11}C_r (2x^2)^{11-r} \left(-\frac{3}{x}\right)^r$$

$$\Rightarrow x_{r+1} = {}^{11}C_r \cdot 2^{11-r} x^{22-2r} (-3)^r x^{-r}$$

$$\Rightarrow x_{r+1} = {}^{11}C_r \cdot 2^{11-r} x^{22-3r} (-3)^r$$

প্রশ্নমতে $22-3r=10$

$$\Rightarrow 3r = 22-10$$

$$\Rightarrow r = 4$$

$$\therefore x^{10} \text{ এর সহগ} = {}^{11}C_4 \cdot 2^{11-4} (-3)^4$$

$$= {}^{11}C_4 \cdot 2^7 \cdot 3^4 \quad (\text{Ans})$$

Math: $\left(\frac{1}{x^2} - x\right)^{18}$ এর বিকৃতিতে x মুক্ত পদের মান নির্ণয় কর।

⇒ এখানে $r+1$ তম পদ, $x_{r+1} = {}^{18}C_r \left(\frac{1}{x^2}\right)^{18-r} (-x)^r$

$$x_{r+1} = {}^{18}C_r x^{-36+2r} (-1)^r x^r$$

$$= {}^{18}C_r \cdot x^{-36+3r} (-1)^r$$

প্রশ্নমতে $-36+3r=0$

$$\Rightarrow 3r = 36$$

$$\Rightarrow r = 12$$

Subject : Date :

$$\therefore x \text{ বর্জিত পদের মান} = {}^{18}C_{12} (-1)^{12} \\ = {}^{18}C_{12}$$

Maths: $(2x + \frac{1}{6x})^{10}$ এর বিস্তৃতিতে x মুক্ত পদের মান নির্ণয় কর।

⇒ এখানে, $r+1$ তম পদ, $x_{r+1} = {}^{10}C_r (2x)^{10-r} (\frac{1}{6x})^r$

$$x_{r+1} = {}^{10}C_r \cdot 2^{10-r} \cdot x^{10-r} \cdot x^{-r} \cdot (\frac{1}{6})^r$$

$$= {}^{10}C_r \cdot 2^{10-r} \cdot \frac{1}{6^r} \cdot x^{10-2r}$$

প্রথমতে $10-2r=0$

$$r=5$$

$$\therefore x \text{ মুক্ত পদের মান} = {}^{10}C_5 2^{10-5} \frac{1}{6^5} x^{10-2 \cdot 5}$$

$$= {}^{10}C_5 \times 32 \times \frac{1}{6^5}$$

Math: $(x+2x)^5$ এর বিস্তৃতিতে x^3 এর সহগ 320 হলে a এর মান নির্ণয় কর।

⇒ এখানে, $(3+1)$ তম পদে x^3 বিদ্যমান।

$$x_{(3+1)} = {}^5C_3 a^{5-3} (2x)^3$$

$$= {}^5C_3 a^2 \times 8x^3$$

Subject :

Date :

প্রসঙ্গত ${}^nC_3 \cdot 8a^2 = 320$

$$\Rightarrow \frac{1 \cdot 2}{1 \cdot 2 \cdot 3} \cdot a^2 = 40$$

$$\Rightarrow \frac{2 \times 4 \times 3 \times 2}{3 \times 2 \times 2 \times 1} \cdot a^2 = 40$$

$$\Rightarrow 10a^2 = 40$$

$$\Rightarrow a^2 = 4$$

$$\Rightarrow a = \pm 2$$

Math: $(3 + \frac{x}{2})^n$ এর বিকৃতিতে x^7 ও x^8 এর সহগদ্বয়
একসত্তর সমান হলে, $n = ?$

⇒ এখানে, $(7+1)$ তম পদে x^7 বিদ্যমান

$$\therefore T_{7+1} = {}^nC_7 \cdot 3^{n-7} \left(\frac{x}{2}\right)^7$$

$$= {}^nC_7 \cdot 3^{n-7} \cdot \frac{1}{2^7} \cdot x^7$$

আবার, $(8+1)$ তম পদে x^8 বিদ্যমান।

$$T_{8+1} = {}^nC_8 \cdot 3^{n-8} \left(\frac{x}{2}\right)^8$$

$$= {}^nC_8 \cdot 3^{n-8} \cdot \frac{1}{2^8} \cdot x^8$$

প্রসঙ্গত $T_{7+1} = T_{8+1}$

Subject :

Date :

প্রদত্ত

$${}^{m-7}C_7 \cdot 3^{m-7} \cdot 2^{-7} = {}^{m-8}C_8 \cdot 3^{m-8} \cdot 2^{-8}$$

$$\Rightarrow \frac{1}{17} \frac{m-7}{m-7} \cdot 3 \cdot 3^{m-8} \cdot 2^{-7} = \frac{1}{18} \frac{m-7}{m-8} \cdot 3^{m-8} \cdot 2^{-8}$$

$$\Rightarrow \frac{3 \cdot 2^{-7}}{17(m-7)(m-8)} = \frac{1}{8 \cdot 17(m-8)} \cdot 2^{-7} \cdot 2^{-1}$$

$$\Rightarrow \frac{3}{m-7} = \frac{1}{16}$$

$$\Rightarrow m-7 = 48$$

$$\Rightarrow m = 55 \text{ (Ans.)}$$

Math: $(1+x)^{44}$ এর বিকৃতিতে 21তম পদ ও 22তম পদ সমান হলে $x = ?$

$$\Rightarrow {}^{44}C_{20+1} = {}^{44}C_{20} \cdot 1^{44-20} \cdot x^{20}$$
$$= {}^{44}C_{20} x^{20}$$

$${}^{44}C_{21+1} = {}^{44}C_{21} \cdot 1^{44-21} \cdot x^{21}$$
$$= {}^{44}C_{21} x^{21}$$

প্রদত্ত, ${}^{44}C_{20} x^{20} = {}^{44}C_{21} x^{21}$

$$\Rightarrow \frac{44}{20} \cdot x^{20} = \frac{44}{21} \cdot x^{21}$$

JANANI

Subject :

Date :

$$\Rightarrow \frac{1}{20 \cdot 24} = \frac{x}{21 \cdot 23}$$

$$\Rightarrow \frac{1}{24 \cdot 23} = \frac{x}{21 \cdot 23}$$

$$\Rightarrow x = \frac{21}{24} \text{ (Ans.)}$$

$$* nC_r = nC_{n-r}$$

Math: $(1+x)^{14}$ এর বিস্তৃতিতে $(1+1)$ কতবার গুণিত
কম অদ্বয়ের সহগদ্বয় সমান হলে $r=?$

$$\Rightarrow x(r+1) = \frac{14}{r} x^{r-1}$$

$$x(r+1) = {}^{14}C_r x^r$$

$$x(3r-1) = {}^{14}C_{(3r-2)+1} x^{3r-2}$$

অতঃপর,

$${}^{14}C_r = {}^{14}C_{(3r-2)+1}$$

$${}^{14}C_r = {}^{14}C_{3r-1}$$

(unsolved)

$${}^{14}C_{14-r} = {}^{14}C_{3r-1}$$

$$\Rightarrow 14-r = 3r-1$$

$$\Rightarrow$$

Subject :

Date :

Math: $(1+x)^n$ এর বিকৃতিতে $(1+x)$ তম পদের সহগ এবং $(1+x)$ তম পদের সহগ সমান হলে, দেখাও যে $2r = n-2$

$$\Rightarrow r+1 \text{ তম পদ, } f(r+1) = {}^nC_{r+1} x^{r+1}$$

$$r+3 = (r+2) + 1 \text{ তম পদ, } f_{(r+2)+1} = {}^nC_{r+2} x^{r+2}$$

$$\text{প্রক্কমতে, } {}^nC_{r+1} = {}^nC_{r+2}$$

$$\Rightarrow {}^nC_{n-r} = {}^nC_{r+2}$$

$$\Rightarrow n-r = r+2$$

$$\Rightarrow 2r = n-2 \quad (\text{Shown})$$

Math: $(a+3x)^n$ এর বিকৃতিতে ১ম তিনটি পদ যথাক্রমে b , $\frac{21}{2}bx$, $\frac{189}{4}bx^2$ হলে, a , b ও n এর মান নিশ্চয় কর।

$$\Rightarrow (a+3x)^n = a^n + {}^nC_1 a^{n-1} (3x) + {}^nC_2 a^{n-2} (3x)^2$$

$$\text{প্রক্কমতে, } a^n = b \quad \text{--- (i)}$$

$${}^nC_1 a^{n-1} \cdot 3x = b \cdot \frac{21}{2} bx$$

$$\Rightarrow \frac{n}{1} a^n \cdot a^{-1} \cdot 3x = \frac{21}{2} a^n x$$

$$\Rightarrow \frac{n}{1} = \frac{21}{2}$$

$$\Rightarrow n = \frac{7a}{2} \quad \text{--- (ii)}$$

JANANI

Subject :

Date :

$$\text{Ans: } {}^nC_2 a^{n-2} \cdot 9x^2 = \frac{189}{4} b x^2$$

$$\Rightarrow \frac{n}{12(n-2)} a^n \cdot a^{-2} = \frac{189}{4 \times 9} a^n$$

$$\Rightarrow \frac{n(n-1)(n-2)}{12(n-2)} = \frac{21}{4} a^2$$

$$\Rightarrow n^2 - n = \frac{21}{2} a^2$$

$$\Rightarrow \frac{49a^2}{4} - \frac{7a}{2} = \frac{21}{2} a^2$$

$$\Rightarrow \frac{49a}{4} - \frac{21}{2} a = \frac{7}{2}$$

$$\Rightarrow a \left(\frac{49-42}{4} \right) = \frac{7}{2}$$

$$\Rightarrow a \cdot \frac{7}{4} = \frac{7}{2}$$

$$\Rightarrow a = 2$$

$$\therefore n = \frac{14}{2} = 7$$

$$b = 2^7 = 128$$

Math: $(1+x)^p (1+\frac{1}{x})^q$ এর বিস্তৃতিতে x বর্জিত পদ নির্ণয় কর।

$$\Rightarrow (1+x)^p (1+\frac{1}{x})^q = (1+x)^p \left(\frac{1+x}{x} \right)^q$$

$$= x^{-q} (1+x)^{p+q}$$

JANANI

Subject :

Date :

যদি $(1+x)$ তম পদে x^9 বিদ্যমান।

$$x^{r+1} = {}^{p+q}C_r x^r$$

প্রথমতে, $r=9$

$$\therefore x^{r+1} = {}^{p+q}C_9 x^9$$

$$\therefore x \text{ বর্নিত পদ} = {}^{p+q}C_9 \quad (\text{Ans.})$$

Math: $(1-x)^8(1+x)^7$ এর বিস্তৃতিতে x^7 এর সহগ কত?

$$\Rightarrow (1-x)^8(1+x)^7 = (1-x)(1-x)^7(1+x)^7$$
$$= (1-x)\{(1-x)(1+x)\}^7$$

$$= 1-x(1-x^2)^7$$

$$= (1-x^2)^7 - x(1-x^2)^7$$

$(1-x^2)^7$ এর বিস্তৃতিতে x^7 পাওয়া সম্ভব নয়।

যদি $(1-x^2)^7$ এর বিস্তৃতিতে $r+1$ তম পদে x^6 বিদ্যমান।

$$\therefore x^{r+1} = {}^7C_r (-x^2)^r$$

$$= {}^7C_r (-1)^r x^{2r}$$

প্রথমতে, $2r=6$

$$r=3$$

JANANI

Subject :

Date :

$$\therefore x^7 \text{ এর সহগ} = {}^7C_7 (-1)^7$$

$$= - {}^7C_7 (-1)^7$$

$$= {}^7C_7 \text{ (Ans.)}$$

~~সহগ~~

সহগের যোগফল

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\text{সহগের যোগফল} = 1 + 2 + 1 = 4 = 2^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\text{সহগের যোগফল} = 1 + 3 + 3 + 1 = 8 = 2^3$$

অর্থাৎ $(a+b)$ এর ঘাত n হলে সহগের যোগফল $= 2^n$

$a=1$ এবং $b=1$ হলে

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\Rightarrow (1+1)^2 = 1 + 2 + 1 = 4$$

$$\Rightarrow 4 = 4 \rightarrow \text{যা সহগের যোগফল}$$

$a=1$ এবং $b=1$ হলে

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(1+1)^3 = 1 + 3 + 3 + 1$$

$$8 = 8 \rightarrow \text{যা সহগের যোগফল}$$

$\therefore (a+2b)^3$ এর বিকৃতিতে সহগগুলোর যোগফল

$$= (1+2)^3 = 3^3 = 27$$

JANANI

Subject :

Date :

Math: $(3x - \frac{1}{2x})^7$ এর বিস্তৃতিতে মধ্যস্থলোর যোগফল কত?

⇒ $x = 1$ হলে

$$(3x - \frac{1}{2x})^7 = (3 - \frac{1}{2})^7 \\ = (\frac{6-1}{2})^7 = (\frac{5}{2})^7$$

∴ মধ্যস্থলোর যোগফল $(\frac{5}{2})^7$

Math: $(a+b+c)^{10}$ এর বিস্তৃতিতে অদমংখ্যা কত?

$$\Rightarrow (a+b+c)^{10} = a^{10} + {}^{10}C_1 a^{10-1} (b+c)^1 + {}^{10}C_2 a^{10-2} (b+c)^2 \\ + {}^{10}C_3 a^{10-3} (b+c)^3 + \dots (b+c)^{10}$$

$$= a^{10}$$

$$\text{অদমংখ্যা} = 1 + 2 + 3 + 4 + 5 + \dots 11$$

$$= \frac{11(11+1)}{2}$$

$$= 66$$

∴ $(a+b+c)^{10}$ এর বিস্তৃতিতে অদমংখ্যা 66।

মধ্যপদ (Middle Term)

$$(a+b)^2 = a^2 + 2ab + b^2 \rightarrow \text{মধ্যপদ } 2ab$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \rightarrow \text{মধ্যপদ } 3a^2b \text{ ও } 3ab^2$$

সেই n হলে অদমংখ্যা $(n+1)$

Subject :

Date :

যদি ছাত্র দ্বারা পদসংখ্যা বিকল্প হয়: সর্বশেষ ১টি

ছাত্র বিদ্যমান হলে পদসংখ্যা দ্বারা তাই সর্বশেষ ২টি।

$$(a+b)^2 = a^2 + 2ab + b^2$$

ছাত্র ২ এর: $2 = \left(\frac{2}{2} + 1\right)$ তম পদ সর্বশেষ।

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

ছাত্র ৫ এর: সর্বশেষ $\left(\frac{5}{2} + 1\right) = 3$ তম পদ।

* ছাত্র দ্বারা হলে $\frac{n}{2} + 1$ তম পদ সর্বশেষ।

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

ছাত্র ৩ এর: সর্বশেষ $\left(\frac{3-1}{2} + 1\right) = 2$ এর: $\left(\frac{3+1}{2} + 1\right) = 3$ তম পদ ছয়।

* ছাত্র বিদ্যমান হলে সর্বশেষ হবে $\left(\frac{n+1}{2} + 1\right)$ এর: $\left(\frac{n-1}{2} + 1\right)$ তম পদ।

Math: $\left(x^2 - 2 + \frac{1}{x^2}\right)^n$ রাশিটির বিস্তৃতিতে সর্বশেষটি নির্ণয় কর।

$$\Rightarrow \left(x^2 - 2 + \frac{1}{x^2}\right)^n = \left(x^2 - 2 \cdot \frac{1}{x} \cdot x + \frac{1}{x^2}\right)^n$$
$$= \left(x - \frac{1}{x}\right)^{2n}$$

এখানে, $\frac{2n}{n} + 1 = n + 1$ তম পদ সর্বশেষ।

Janani

Subject : _____ Date : _____

$$\therefore \text{সর্বপদ, } x_{(n+1)} = {}^{2n}C_n x^{2n-n} \left(-\frac{1}{x}\right)^n$$

$$= {}^{2n}C_n x^n (-1)^n \frac{1}{x^n}$$

$$= {}^{2n}C_n (-1)^n \quad (\text{Ans.})$$

$$2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 = (2 \times 1)(2 \times 2)(2 \times 3)(2 \times 4)(2 \times 5)$$

$$= 2^5 \times 1 \times 2 \times 3 \times 4 \times 5$$

$$= 2^5 \cdot 120$$

$$\text{সমস্ত পদগুণিত, } 2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \dots 2^n$$

$$= (2 \times 1)(2 \times 2)(2 \times 3)(2 \times 4)(2 \times 5) \dots 2^n \cdot 2^n$$

$$= 2^n (1 \times 2 \times 3 \times 4 \times 5 \dots n)$$

$$= 2^n \cdot n!$$

Math: দেখাও যে $\left(x - \frac{1}{x}\right)^{2n}$ এর বিস্তৃতিতে সর্বপদ

$$\frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{n!} (-2)^n$$

$\Rightarrow$ এখানে $\left(x - \frac{1}{x}\right)^{2n}$ এর বিস্তৃতিতে $\left(\frac{2n}{2} + 1\right) = n+1$ তম পদ সর্বপদ।

$$\therefore x_{(n+1)} = {}^{2n}C_n x^{2n-n} \left(-\frac{1}{x}\right)^n$$

$$= {}^{2n}C_n x^n (-1)^n \frac{1}{x^n}$$

$$= {}^{2n}C_n (-1)^n$$

Subject :

Date :

$$\begin{aligned}
 x(n+1) &= \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdots 2n}{n \cdot n} (-1)^n \\
 &= \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdots 2n}{n \cdot n} (-1)^n \\
 &= \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1) \cdot (2 \cdot 4 \cdot 6 \cdots 2n)}{n \cdot n} (-1)^n \\
 &= \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) \cdot 2^n \cdot n}{n \cdot n} (-1)^n \\
 &= \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) \cdot 2^n (-1)^n}{n} \\
 &= \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) (-2)^n}{n} \quad (\text{Shown})
 \end{aligned}$$

Math: $\left(\frac{x}{y} + \frac{y}{x}\right)^{21}$ এর বিকল্পের মধ্য অদ্বয় নির্ণয় কর।

$$\Rightarrow \text{২ম মধ্য অদ্বয়} = \frac{21-1}{2} + 1 = 11 \text{ম অদ্বয়}$$

$$\begin{aligned}
 x(10+1) &= {}^{21}C_{10} \left(\frac{x}{y}\right)^{21-10} \left(\frac{y}{x}\right)^{10} \\
 &= {}^{21}C_{10} \frac{21!}{y^{11}} \cdot \frac{y^{10}}{x^{10}} \\
 &= {}^{21}C_{10} \frac{x}{y}
 \end{aligned}$$

Subject :

Date :

$$2\text{য় সঙ্খ্যপদ} = \frac{21+1}{2} + 1 = (11+1) \text{ তম পদ}$$

$$T_{(11+1)} = {}^{21}C_{11} \left(\frac{x}{y}\right)^{21-11} \left(\frac{y}{x}\right)^{11}$$

$$= {}^{21}C_{11} \left(\frac{x}{y}\right)^{10} \left(\frac{y}{x}\right)^{11}$$

$$= {}^{21}C_{11} \frac{y}{x}$$

Math: $\left(2x + \frac{1}{6x}\right)^6$ এবং $\left(x + \frac{1}{3x}\right)^{2n}$ এর বিস্তৃতিতে
সঙ্খ্যপদসমূহ সমান হলে $n = ?$

⇒ $\left(2x + \frac{1}{6x}\right)^6$ এর বিস্তৃতিতে $\left(\frac{6}{2} + 1\right) = (3+1)$ তম
পদ সঙ্খ্যপদ।

$$\therefore T_{(3+1)} = {}^6C_3 (2x)^{6-3} \left(\frac{1}{6x}\right)^3$$

$$= {}^6C_3 \cdot 2^3 \cdot x^{+3} \frac{1}{6^3 x^3}$$

$$= {}^6C_3 \left(\frac{2}{6}\right)^3$$

$$= {}^6C_3 \cdot 3^{-3}$$

$\left(x + \frac{1}{3x}\right)^{2n}$ এর বিস্তৃতিতে $\left(\frac{2n}{2} + 1\right) = n+1$ তম পদ
সঙ্খ্যপদ।

$$\therefore T_{(n+1)} = {}^{2n}C_n x^{2n-n} \left(\frac{1}{3x}\right)^n$$

$$= {}^{2n}C_n x^n 3^{-n} x^{-n}$$

Jannani

Subject : Date :

$$\therefore f(n+1) = {}^{2n}C_n 3^{-n}$$

প্রসঙ্গতঃ ${}^{2n}C_n 3^{-n} = {}^6C_3 3^{-3}$

$$\Rightarrow 3^{-n} = 3^{-3}$$

$$\Rightarrow n = 3$$

উদাহরণ : $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \dots + C_nx^n$ হলে
প্রমাণ কর যে, $C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}$

দেওয়া আছে,

$$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \dots + C_nx^n$$

$x = 1$ বসিয়ে পাই

$$2^n = C_0 + C_1 + C_2 + C_3 + \dots + C_n \quad \text{--- (i)}$$

$x = -1$ বসিয়ে পাই

$$C_0 - C_1 + C_2 - C_3 + C_4 - \dots + C_n = 0 \quad \text{--- (ii)}$$

$$\Rightarrow C_0 + C_2 + C_4 = C_1 + C_3 + C_5 + \dots$$

$$\Rightarrow C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots$$

(Proved)

i + ii $\Rightarrow$

$$2C_0 + 2C_2 + 2C_4 + \dots = 2^n$$

$$\Rightarrow C_0 + C_2 + C_4 + \dots = 2^n \cdot 2^{-1}$$

$$= 2^{n-1}$$

$$\Rightarrow C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1} \quad \text{(Proved)}$$

Subject :

Date :

$$\begin{aligned}
 (a+b)^3 &= (a+b)(a^2+2ab+b^2) \\
 &= a^3 + 2a^2b + ab^2 \\
 &\quad + a^2b + 2ab^2 + b^3 \\
 &= a^3 + 3a^2b + 3ab^2 + b^3
 \end{aligned}$$

$$\begin{array}{cccccccc}
 & & & & 1 & & & (a+b)^0 \\
 & & & & 1 & & 1 & (a+b)^1 \\
 & & & 1 & & 2 & & 1 & (a+b)^2 \\
 & & 1 & & 3 & & 3 & & 1 & (a+b)^3 \\
 & 1 & & 6 & & 6 & & 1 & & (a+b)^4 \\
 & 1 & 5 & & 10 & & 10 & & 5 & & 1 & (a+b)^5 \\
 & 1 & 6 & 15 & & 20 & & 15 & 6 & 1 & & (a+b)^6 \\
 1 & 7 & 21 & & 35 & & 35 & & 21 & 7 & 1 & (a+b)^7
 \end{array}$$

$$\begin{aligned}
 (a+b)^7 &= a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 \\
 &\quad + 21a^2b^5 + 7ab^6 + b^7
 \end{aligned}$$

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b^1 + {}^nC_2 a^{n-2}b^2 + {}^nC_3 a^{n-3}b^3 + \dots + b^n; \text{ n शिनाङ्क पूर्ण संख्या }$$

$${}^nC_1 = n$$

JANANI

Subject :

Date :

$$nC_2 = \frac{n(n-1)}{2}$$

$$nC_3 = \frac{n(n-1)(n-2)}{3}$$

$$\therefore (a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3}a^{n-3}b^3 + \dots b^n; n \in \mathbb{R}$$

এখানে $a=1$ ও $b=x$ বসিয়ে পাই

$$(1+x)^n = a^n 1 + na^{n-1}x + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{3}x^3 + \dots x^n$$

$(a+b)^n$ এর বিস্তৃতিতে $(n+1)$ টি পদ থাকবে। কেন?

$$\Rightarrow (a+b)^3 = a^3 + 3a^2b + \frac{3(3-1)}{2}a^{3-2}b^2 +$$

$$\frac{3(3-1)(3-2)}{3}a^{3-3}b^3 + \frac{3(3-1)(3-2)(3-3)}{4}a^{3-4}b^4 + \frac{3(3-1)(3-2)(3-3)(3-4)}{5}a^{3-5}b^5$$

$$\Rightarrow (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 + 0 + 0 + 0 + 0$$

$$\Rightarrow (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

অর্থাৎ, $(a+b)^3$ এর বিস্তৃতিতে ৪ম চারটি পদ বাদে বাকি সব পদ ০।

JANANI

Subject :

Date :

$$(a+b)^{\frac{1}{2}} = a^{\frac{1}{2}} + \frac{1}{2} a^{\frac{1}{2}-1} b + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} a^{\frac{1}{2}-2} b^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-1)}{3!} b^3 + \dots \dots \infty$$

সুতরাং n বিনামূলি পূর্ণসংখ্যা হলে সীরাটি হবে :
সমীম।

n বিনামূলি পূর্ণসংখ্যা না হলে সীরাটি হবে অসীম।

$$(1-x)^{-1} = 1 + (-1)x^{-1-1}(-x) + \frac{(-1)(-1-1)}{2!} 1^{-1-2} x^2 + \frac{(-1)(-1-1)(-1-2)}{3!} 1^{-1-3} x^3 + \dots$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots \dots \text{upto } \infty ; |x| < 1$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \dots \dots \text{upto } \infty ; |x| < 1$$

$$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots \dots \text{upto } \infty ; |x| < 1$$

$$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots \dots \text{upto } \infty ; |x| < 1$$

$$(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \dots \dots \text{upto } \infty ; |x| < 1$$

$$(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \dots \dots \text{upto } \infty ; |x| < 1$$

Maths বিস্তারিত : $(1-x)^{-1} ; |x| > 1$

JANANI

Subject :

Date :

$$\Rightarrow (1-x)^{-1} = \left\{x\left(\frac{1}{x}-1\right)\right\}^{-1}$$
$$= -x^{-1}\left(1-\frac{1}{x}\right)^{-1}$$

দেওয়া আছে $|x| > 1$

$$\Rightarrow \frac{1}{|x|} < 1$$

$$\therefore \frac{1}{x}\left(1-\frac{1}{x}\right)^{-1} = -\frac{1}{x}\left(1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots \infty\right) \text{ (Ans.)}$$

Ex 10. বিস্তারিত করে নির্ণয় কর এবং বিস্তৃত কর। $\frac{1}{\sqrt[3]{8-3x}}$

$$\Rightarrow \frac{1}{\sqrt[3]{8-3x}} = \frac{1}{(8-3x)^{\frac{1}{3}}}$$

$$= 8^{-\frac{1}{3}} \left(1 - \frac{3x}{8}\right)^{-\frac{1}{3}}$$

$$= \frac{1}{2} \left(1 - \frac{3x}{8}\right)^{-\frac{1}{3}}$$

প্রশ্নসমূহে

$$\left|\frac{3x}{8}\right| < 1$$

$$\Rightarrow -1 < \frac{3x}{8} < 1$$

$$\Rightarrow -\frac{8}{3} < x < \frac{8}{3} ; \text{ যা নির্ণয় করে}$$

$$\text{এখন } \frac{1}{2} \left(1 - \frac{3x}{8}\right)^{-\frac{1}{3}} = 1 + \left(-\frac{1}{3}\right)\left(\frac{-3x}{8}\right) + \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!} \left(\frac{-3x}{8}\right)^2$$
$$+ \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!} \left(\frac{-3x}{8}\right)^3 + \dots \text{ upto } \infty$$

JANANI

Subject :

Date :

$$\Rightarrow \frac{1}{2} \left(1 - \frac{3x}{8}\right)^{-\frac{1}{3}} = \frac{1}{2} \left(1 + \frac{x}{8} + \frac{5}{24}x^2 + \dots \infty\right) \quad (\text{Ans.})$$

Math: $y = x + x^2 + x^3 + \dots \infty$ ২ম (দেখাও য়) $x = y - y^2 + y^3 - y^4 + \dots \infty$

$\Rightarrow$ দেখাও য়াচ্ছে $y = x + x^2 + x^3 + \dots \infty$

$$\Rightarrow 1 + y = 1 + x + x^2 + x^3 + \dots \infty$$

$$\Rightarrow 1 + y = (1 - x)^{-1}$$

$$\Rightarrow (1 + y)^{-1} = 1 - x$$

$$\Rightarrow 1 - x = 1 + y + y^2 + y^3 + y^4 + \dots \infty$$

$$\Rightarrow (1 - x) = 1 - y + y^2 - y^3 + y^4 + \dots \infty$$

$$\Rightarrow x = y - y^2 + y^3 - y^4 + \dots \infty \quad (\text{Shown})$$

Math: $y = x - x^2 + x^3 - x^4 + \dots \infty$ ২ম x ৩য়
জাতির উপক্রমিক ধারায় প্রকাশ কর।

$\Rightarrow$ দেখাও য়াচ্ছে

$$y = x - x^2 + x^3 - x^4 + \dots \infty$$

$$\Rightarrow -y = -x + x^2 - x^3 + x^4 + \dots \infty$$

$$\Rightarrow 1 - y = 1 - x + x^2 - x^3 + x^4 + \dots \infty$$

$$\Rightarrow 1 - y = (1 + x)^{-1}$$

$$\Rightarrow (1 - y)^{-1} = 1 + x$$

$$\Rightarrow 1 + x = 1 + y + y^2 + y^3 + \dots \infty$$

$$\Rightarrow x = y + y^2 + y^3 + \dots \infty \quad (\text{Ans.})$$

Subject :

Date :

Maths $y = 2x + 3x^2 + 4x^3 + \dots \infty$ $2x$ $3x^2$ $4x^3$ $5x^4$ $6x^5$ $7x^6$ $8x^7$ $9x^8$ $10x^9$ $11x^{10}$ $12x^{11}$ $13x^{12}$ $14x^{13}$ $15x^{14}$ $16x^{15}$ $17x^{16}$ $18x^{17}$ $19x^{18}$ $20x^{19}$ $21x^{20}$ $22x^{21}$ $23x^{22}$ $24x^{23}$ $25x^{24}$ $26x^{25}$ $27x^{26}$ $28x^{27}$ $29x^{28}$ $30x^{29}$ $31x^{30}$ $32x^{31}$ $33x^{32}$ $34x^{33}$ $35x^{34}$ $36x^{35}$ $37x^{36}$ $38x^{37}$ $39x^{38}$ $40x^{39}$ $41x^{40}$ $42x^{41}$ $43x^{42}$ $44x^{43}$ $45x^{44}$ $46x^{45}$ $47x^{46}$ $48x^{47}$ $49x^{48}$ $50x^{49}$ $51x^{50}$ $52x^{51}$ $53x^{52}$ $54x^{53}$ $55x^{54}$ $56x^{55}$ $57x^{56}$ $58x^{57}$ $59x^{58}$ $60x^{59}$ $61x^{60}$ $62x^{61}$ $63x^{62}$ $64x^{63}$ $65x^{64}$ $66x^{65}$ $67x^{66}$ $68x^{67}$ $69x^{68}$ $70x^{69}$ $71x^{70}$ $72x^{71}$ $73x^{72}$ $74x^{73}$ $75x^{74}$ $76x^{75}$ $77x^{76}$ $78x^{77}$ $79x^{78}$ $80x^{79}$ $81x^{80}$ $82x^{81}$ $83x^{82}$ $84x^{83}$ $85x^{84}$ $86x^{85}$ $87x^{86}$ $88x^{87}$ $89x^{88}$ $90x^{89}$ $91x^{90}$ $92x^{91}$ $93x^{92}$ $94x^{93}$ $95x^{94}$ $96x^{95}$ $97x^{96}$ $98x^{97}$ $99x^{98}$ $100x^{99}$ $101x^{100}$ $102x^{101}$ $103x^{102}$ $104x^{103}$ $105x^{104}$ $106x^{105}$ $107x^{106}$ $108x^{107}$ $109x^{108}$ $110x^{109}$ $111x^{110}$ $112x^{111}$ $113x^{112}$ $114x^{113}$ $115x^{114}$ $116x^{115}$ $117x^{116}$ $118x^{117}$ $119x^{118}$ $120x^{119}$ $121x^{120}$ $122x^{121}$ $123x^{122}$ $124x^{123}$ $125x^{124}$ $126x^{125}$ $127x^{126}$ $128x^{127}$ $129x^{128}$ $130x^{129}$ $131x^{130}$ $132x^{131}$ $133x^{132}$ $134x^{133}$ $135x^{134}$ $136x^{135}$ $137x^{136}$ $138x^{137}$ $139x^{138}$ $140x^{139}$ $141x^{140}$ $142x^{141}$ $143x^{142}$ $144x^{143}$ $145x^{144}$ $146x^{145}$ $147x^{146}$ $148x^{147}$ $149x^{148}$ $150x^{149}$ $151x^{150}$ $152x^{151}$ $153x^{152}$ $154x^{153}$ $155x^{154}$ $156x^{155}$ $157x^{156}$ $158x^{157}$ $159x^{158}$ $160x^{159}$ $161x^{160}$ $162x^{161}$ $163x^{162}$ $164x^{163}$ $165x^{164}$ $166x^{165}$ $167x^{166}$ $168x^{167}$ $169x^{168}$ 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Subject :

Date :

Math: $(1 - \frac{x}{6})^{\frac{1}{2}}$ কে x এর টেইলর সিরিজ দ্বারা প্রকাশ
 করে লম্বা বিস্তারিত কর/এবং তা থেকে দেখাও যে, $1 - \frac{1}{6} - \frac{1}{6} \cdot \frac{1}{12} - \frac{1}{6} \cdot \frac{1}{12} \cdot \frac{1}{18} - \dots = \sqrt{\frac{2}{3}}$

$$\Rightarrow (1 - \frac{x}{6})^{\frac{1}{2}} = 1 + \frac{1}{2}(\frac{-x}{6}) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\frac{-x}{6})^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(\frac{-x}{6})^3 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)(\frac{1}{2}-3)}{4!}(\frac{-x}{6})^4 + \dots$$

প্রমাণে

$$(1 - \frac{x}{6})^{\frac{1}{2}} = \sqrt{\frac{2}{3}}$$

$$\Rightarrow 1 - \frac{x}{6} = \frac{2}{3} + \dots$$

$$\Rightarrow \frac{x}{6} = \frac{1}{3} \Rightarrow x = 2$$

$(1 - \frac{x}{6})^{\frac{1}{2}}$ এর বিস্তারিত $x=2$ বসিয়ে পাই

$$1 + \frac{1}{2}(\frac{-2}{6}) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(\frac{-2}{6})^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(\frac{-2}{6})^3 + \dots = (1 - \frac{2}{6})^{\frac{1}{2}}$$

$$= 1 - \frac{1}{6} - \frac{1}{6} \cdot \frac{1}{12} - \frac{1}{6} \cdot \frac{1}{12} \cdot \frac{1}{18} = \sqrt{\frac{2}{3}}$$

(Shown)

Subject :

Date :

ମାଟ୍ରିକ୍ସ

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2} x^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} x^3 + \dots$$

$\downarrow$
(2+1)ତମ ଅଂଶ

$\downarrow$
(3+1)ତମ ଅଂଶ

$$\therefore r+1 \text{ ତମ ଅଂଶ, } x(r+1) = \frac{n(n-1)(n-2) \dots \{n-(r-1)\}}{1 \cdot 2 \cdot 3 \dots r} x^r$$

$$\therefore x(r+1) = \frac{n(n-1)(n-2) \dots (n-r+1)}{1 \cdot 2 \cdot 3 \dots r} x^r$$

Maths ଦିଆଯାଇଛି $(1-4x)^{-\frac{1}{2}}$ ଏବଂ ବିକାଶିତ x^{π} ଏବଂ

$$\text{ମଞ୍ଜୁ} = \frac{(2\pi)!}{(\pi!)^2}$$

$$\Rightarrow \pi+1 \text{ ତମ ପଦ } x^{\pi+1} = \frac{-\frac{1}{2}(-\frac{1}{2}-1)(-\frac{1}{2}-2)\dots[-\frac{1}{2}-(\pi-1)]}{\pi!} (-4)^{\pi} x^{\pi}$$

$$x^{\pi+1} = \frac{-\frac{1}{2}(-\frac{3}{2})(-\frac{5}{2})\dots(\frac{1-2\pi+2}{2})}{\pi!} (-4)^{\pi} x^{\pi}$$

$$\therefore x^{\pi} \text{ ଏବଂ ମଞ୍ଜୁ} = \frac{-\frac{1}{2}(-\frac{3}{2})(-\frac{5}{2})\dots(\frac{1-2\pi}{2})}{\pi!} (-4)^{\pi}$$

$$= \frac{(-1)^{\pi} \cdot 1 \cdot 3 \cdot 5 \cdot (2\pi-1)}{2^{\pi} \pi!} (-4)^{\pi}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \dots 2\pi}{2^{\pi} \pi! (2 \cdot 4 \cdot 6 \dots 2\pi)} 4^{\pi}$$

$$= \frac{2\pi}{2^{\pi} \pi! \cdot 2^{\pi} \pi!} 2^{2\pi}$$

$$= \frac{2\pi}{2^{2\pi} (\pi!)^2}$$

$$= \frac{(2\pi)!}{(\pi!)^2} \quad (\text{Showed})$$

Subject :

Date :

Math: $\frac{1}{(1-x)(1-2x)}$ এর বিকৃতিতে x^n এর সহগ

নির্ণয় কর।

$$\Rightarrow \frac{1}{(1-x)(1-2x)} = \frac{1}{(1-x)(-1)} + \frac{1}{(1-2x) \frac{1}{2}}$$

$$= -(1-x)^{-1} + 2(1-2x)^{-1}$$

$$= 2(1-2x)^{-1} - (1-x)^{-1}$$

$$= 2(1 + (2x)^2 + (2x)^3 + (2x)^4 + \dots) - (1 + x + x^2 + \dots)$$

$$\therefore x^n \text{ এর সহগ} = 2 \cdot 2^n - 1$$

$$= 2^{n+1} - 1 \quad (\text{Ans.})$$

Math: $\frac{1}{1-5x+6x^2}$ এর বিকৃতিতে x^n এর সহগ

নির্ণয় কর।

$$\Rightarrow 1-5x+6x^2 = 6x^2-5x+1$$

$$= 6x^2-3x-2x+1$$

$$= 3x(2x-1) - 1(2x-1)$$

$$= (3x-1)(2x-1)$$

$$= (1-3x)(1-2x)$$

$$\text{এখন, } \frac{1}{(1-3x)(1-2x)} = \frac{1}{(1-3x) \frac{1}{3}} + \frac{1}{(1-2x) \frac{1}{2}}$$

JANANI

Subject :

Date :

$$\Rightarrow \frac{1}{(1-2x)(1-3x)} = 3(1-3x)^{-1} - 2(1-2x)^{-1}$$

$$= 3(1 + 3x + (3x)^2 + \dots \infty) - 2[1 + (2x) + (2x)^2 + \dots \infty]$$

$$\therefore x^{10} \text{ এর সহগ} = 3 \cdot 3^{10} - 2 \cdot 2^{10}$$

$$= 3^{11} - 2^{11} \quad (\text{Ans.})$$

ধারার বৈশিষ্ট্য

* দ্বিতীয় ধারার বৈশিষ্ট্য :

- (i) ১ দিয়ে শুরু হয়।
- (ii) ভগ্নাংশ অমল বিহীন থাকে।
- (iii) Gradually হ্রাস ও লবের Entry বাড়তে থাকে।

Math: $1 + \frac{1}{3} + \frac{1 \cdot 2}{3 \cdot 6} + \frac{1 \cdot 2 \cdot 3}{3 \cdot 6 \cdot 9} + \dots \infty$ ধারাটির
যোগফল নির্ণয় কর।

⇒ ধরি

$$1 + \frac{1}{3} + \frac{1 \cdot 2}{3 \cdot 6} + \frac{1 \cdot 2 \cdot 3}{3 \cdot 6 \cdot 9} + \dots \infty = (1+x)^n$$

$$\text{এখন } (1+x)^n = 1 + nx + \frac{n(n-1)}{2} x^2 + \frac{n(n-1)(n-2)}{6} x^3 + \dots \infty$$

$$\therefore nx = \frac{1}{3} \quad \text{--- (i)}$$

$$\frac{n(n-1)}{2} = \frac{3}{3 \cdot 6} \quad \text{--- (ii)}$$

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Subject :

Date :

ii) ধরে নেওয়া

$$\frac{n(n-1)}{2} \times 2 = \frac{1}{6}$$

$$\Rightarrow \frac{n(n-1)}{2} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{2} (1 - n) = \frac{1}{6}$$

$$\Rightarrow \frac{1}{2} - n = \frac{1}{3}$$

$$\Rightarrow n = 1 - \frac{1}{3} =$$

$$\Rightarrow n = \frac{1}{3} - 1$$

$$= \frac{-2}{3}$$

$$\therefore n = \frac{1}{3} \times \frac{2}{-2} = -\frac{1}{2}$$

$$\text{নির্ণেয় ধারার যোগফল} = \left\{ 1 + \left(-\frac{2}{3} \right) \right\}^{-\frac{1}{2}} \quad (\text{Ans})$$

$$\text{Math: } 1 + \frac{1}{4} + \frac{1 \cdot 3}{4 \cdot 8} + \frac{1 \cdot 3 \cdot 5}{4 \cdot 8 \cdot 12} + \dots \infty$$

সামান্তরিক যোগফল নির্ণয় কর।

$$\Rightarrow \text{ধরি } 1 + \frac{1}{4} + \frac{1 \cdot 3}{4 \cdot 8} + \frac{1 \cdot 3 \cdot 5}{4 \cdot 8 \cdot 12} + \dots \infty = (1+x)^n$$

Subject : _____ Date : _____

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2} x^2 + \dots \infty$$

$$\therefore nx = \frac{1}{4} \quad \text{--- (i)}$$

$$\frac{n(n-1)}{2} x^2 = \frac{3}{32} \quad \text{--- (ii)}$$

→ (ii) શરૂ બારે

$$\frac{nx(n-1)x}{2} = \frac{3}{32}$$

$$\Rightarrow \frac{\frac{1}{4} \left(\frac{1}{4} - x \right)}{2} = \frac{3}{32}$$

$$\Rightarrow \frac{1}{4} - x = \frac{6}{8} = \frac{3}{4}$$

$$\Rightarrow x = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2}$$

$$\therefore n = \frac{1}{4} \times -2$$
$$= -\frac{1}{2}$$

$$\therefore \text{જાણક યોગશક્તિ} = \left(1 - \frac{1}{2}\right)^{-\frac{1}{2}} \text{ (Ans.)}$$